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MATTER, ENERGY, FORCE, AND WORK

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MATTER, ENERGY, FORCE AND WORK

A PLAIN PRESENTATION OF FUNDAMENTAL PHYSICAL CONCEPTS AND OF THE VORTEX-ATOM AND OTHER THEORIES

BY

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TO THE

Massachusetts Institute of Technology,

TO THE MEMBERS, PAST AND PRESENT, OF ITS CORPORATION AND FACULTY, AND TO ITS ALUMNI,

AND BECAUSE OF WHAT, INDIVIDUALLY AND COLLECTIVELY, THEY HAVE BROUGHT INTO MY LIFE,

I Dedicate

WHATEVER OF GOOD THIS VOLUME MAY CONTAIN.

PREFACE.

THE aim of this book, prepared during a period of enforced quiet, is to present in a plain and logical manner some fundamental ideas and definitions of physics. The purpose is not to set forth the experimental side of the subject, nor to describe phenomena or laws; the intention is rather to assume a slight knowledge of these, and to proceed in an orderly manner to develop the concepts and definitions.

Engineers and members of the other technical professions will find here, it is hoped, an aid to clearer thinking in their practical dealing with the subjects treated. Now, as never before, correct views of energy, force, and work are essential in these professions, through the progress of the application of science to the industrial arts. The extent to which these applications are daily brought home to the untechnical members of the community is likewise so great that acquaintance with their underlying ideas is no less a matter of self-interest than of education. The book naturally addresses itself most directly, however, to students and teachers in physics and chemistry, whether elementary or advanced, and of the natural sciences in general.

To have its greatest educational value, physics must be presented in logical sequence. The fundamental definitions must follow each other in such a manner that each involves but one important idea or proposition in advance of the preceding one. Every term must be clearly defined before being used, and no familiar word should be employed in a

technical sense without previous explanation. Only in this way will the student be assured that at every stage of his progress he possesses a secure and stable body of knowledge, and is not working in a circle. Benefit will be received from the logical habit of thought thus engendered, not less than from the knowledge acquired.

The first of the two parts into which the work has been divided contains its subject-matter proper. The definitions and views given are not designed to be an exposition of current and accepted usage or literature. Rather they constitute a sporadic attempt at clear, consecutive setting forth of individual thought. The book is constructive in spirit, not critical; and it is purposely almost devoid of historical and personal allusions.

Part second consists of summaries of the chief theories of the nature of matter, energy, and force. These are intended mainly to give more concreteness to the concepts than could properly be introduced into the first part. Their purely hypothetical character demands this clear demarcation, but by no means debars them from service in the present connection.

The publication of the volume has been rendered possible by the generous kindness of Mr. Francis Blake, of Boston. The possibility of its preparation, as well as of all other work during several years, has been due to the personal devotion, care, and counsel of Mrs. Holman, and to her coöperation, which has included, as a part, the reading to me of all references consulted and of all the manuscript and proofs for revision. Full acknowledgment is tendered to Lord Kelvin and to Professor J. J. Thomson for permitting the use of extracts from personal letters; to Miss Catharine B. Runkle for friendly aid and criticism; and to my friends Professors H. M. Goodwin and A. A. Noyes for criticism and suggestion.

SILAS W. HOLMAN.

BOSTON, August, 1898.

CONTENTS.

PART I.

CHAPTER I.

SUBSTANCE, MATTER.

CHAPTER II.

MOTION.

CHAPTER III.

ENERGY.

CHAPTER IV.

FORMS OF ENERGY.

CHAPTER V.

Force.

CHAPTER VI.

Kinetic Energy, Kinergety.

CHAPTER VII.

Energy (Continued).

CHAPTER VIII.

Force Measurement.

CHAPTER IX.

WORK.

CHAPTER X.

POTENTIAL ENERGY.

CHAPTER XI

MATTER.

CHAPTER XII.

Comments on Certain Definitions.

PART II.

SPECULATIONS ON MATTER AND ENERGY.

CHAPTER I.

Function of Theory and Hypothesis.

CHAPTER II.

Kinetic Theory of Gases.

CHAPTER III.

Le Sage's Theory of Gravitation.

CHAPTER IV.

The Vortex-Atom Theory.

CHAPTER V.

Nature of Energy and Matter.

Part I.

FUNDAMENTAL PHYSICAL CONCEPTS.

"What is mind?" No matter.
What is matter? Never mind."

attributed to T. H. Key.

CHAPTER I.

SUBSTANCE, MATTER.

Substance, Body, Object. — It may be accepted as a starting-point in physical inquiry that our knowledge of things external to ourselves, as well as of our own bodies, is based upon impressions produced upon the brain by the several senses. These impressions are interpreted by the aid of Reason, thus becoming ideas, or giving rise to ideas and knowledge.

We thus observe that different portions of the space about us affect our senses differently, and that these diverse characteristics are more or less permanently associated with the same portions of space. The apparent behavior of the lower animals shows that they are similarly affected.

Further, we observe that various portions of space possess the power not only to affect our senses, but also to affect one another (using *power* in its primary sense of *ability to do*); that is, we discover changes taking place simultaneously in two or more portions of space, changes which appear to stand to one another in the relation which we commonly denote by the terms *cause* and *effect*.

The direct result of these primary observations is, then, that the space about us may be marked off into definitely bounded portions, each of which is characterized by its own group of powers. The operations and nature of these powers form the chief part of the study of physics. These powers are also called *properties*. Also, in general, any one of these groups may be [illegible] to the othe[illegible]

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of the space occupied by the group sometimes undergoing change during the changing of position.

To indicate what is meant by the "powers" or "properties" referred to, a few of them may be specifically mentioned. It is observed that any portion of space which can be in any way distinctly recognized, possesses the power, when in motion relatively to another recognizable portion, of imparting motion to that other portion, always sacrificing some of its own speed in so doing. It is found that any portion appears to possess the power of approaching that other portion called the earth; that is, that it will approach unless obviously prevented by the intervention of the power of some other portion. This power we know familiarly as gravitation or weight. Many portions have the power of resisting a pull tending to separate them: cohesion. Some portions not only resist any attempt to change their form, but also will return to their original form when again left to themselves: a power called elasticity. Certain portions when brought into contact, are found to undergo spontaneously an entire change of general appearance and of many of their powers: this is known as chemical action and is ascribed to a corresponding power vaguely termed chemical affinity. Without further description, these powers may be enumerated. They are nine in number; viz., power due to motion; power of gravitation; heat power; elastic power; cohesive (and adhesive) power; chemical power; electrical power; magnetic power; and finally power in the form in which electromagnetic action, "light," and "radiant heat" are propagated through space. This list includes all the "powers" properly so called, but they are probably not all distinct from each other. The classification is merely a provisional one, based on the customary grouping of physical phenomena. There are other so-called powers, but they are either different phases of the foregoing, or are not active powers but passive characteristics which might better be

called by another name. It might perhaps be more logical at this stage of the presentation of the subject, to remain content with regarding these groups as mere collocations of powers or properties. This abstract concept is however quite repugnant to the habit of thought engendered by the entire training of early life, and is not in accord with the traditions of physics. Instead of this we are accustomed to regard these groups of powers as constituting attributes of something which occupies, not necessarily wholly filling, the particular portions of space considered, and which has a more or less permanent existence outside of and independent of our minds. We therefore proceed to infer that there is in space something which is endued with these powers. To this we give the name of *substance*. We might then define substance as that which is inferred as existing in space, and as endued with powers to affect the human senses and portions of itself. Assuming however that we may exclude from purely physical definitions any allusion to sensation, since we have recognized at the outset that it is only through sensation that we are aware of physical action, this formula may be reduced to:—

SUBSTANCE IS THAT WHICH IS INFERRED AS EXISTING IN SPACE, AND AS ENDUED WITH POWERS TO AFFECT PORTIONS OF ITSELF.

This definition makes no assumption as to the possibility or impossibility of the entire removal of all of any one power from any portion of substance, nor as to the possibility of the existence of any of the powers apart from substance. Neither does the definition affirm or deny that substance fills any portion of space. Whether it does so is a topic for later inquiry; and whether existence without the occupancy of space is conceivable, is a metaphysical question with which we are not now concerned.

Indeed it may well be remarked here that psychological

and philosophical considerations as to the interpretation to be placed upon such terms as existence and reality form no part of physics. The terms of this science, if consistent with one another, express a definite body of knowledge. Whatever interpretation, from that of Berkeley to that of the materialist, be put upon this knowledge, this interpretation involves ideas outside of the domain of physics, and the same is true of the other natural sciences. The student of these sciences cannot bear too clearly in mind this fact. Always looking forward to an ultimate adequate interpretation of the relation of the mental to the physical, he may rest in the assurance that this interpretation will not vitiate any real physical science, but will only illuminate it.

A Body is any Designated Portion of Substance.

The boundary of a body may be and very often is wholly imaginary. The size may be anything whatever, so long as the characteristics of substance remain. When the boundary is natural and not imaginary, the body may be called *an object.*

Properties. — By quantitative examination it is easily discoverable that some or all of the powers of a body undergo change in amount from time to time. These changes accompany change of motion, change of position, action of one body on another, and so on. Thus if a body in motion imparts to another some of its motion, as through collision, it has less power than before to change the motion of other bodies. A body raised above the surface of the earth appears to have the power to accelerate its own motion towards the earth, or to move other bodies, and it gains in this power as it is raised higher. A hot body has the power of imparting heat to other bodies, and thus of producing such effects as moving a railway train. These few instances

illustrate not only the change in amount of power resident on a body, but also the possibility of its transference from one to another.

Further, it appears that under the same conditions a given body can or does take up always the same amount of a given power. Thus a body moving with the same velocity at different times, possesses the same amount of power to move other bodies. A body acquires the same amount of power in being raised through the same vertical distance at different times. A stated body always emits the same amount of heat in cooling through the same temperature interval. These examples, being in advance of the present stage of our subject, are cited only for the sake of adding a touch of concreteness to the idea of "capacity" next to be introduced.

Ability to receive, carry, or hold the "powers" is implied in the definition of substance, — that is, a receptivity or capacity. But the cases just noted show that besides possessing capacity in this abstract sense, each portion of substance has a quantitatively definite and limited capacity for each (although perhaps not for every) power. The term *capacity* will be employed primarily with this meaning. There are also other characteristics of substance commonly designated as capacities, but they are all related to the powers in ways which lead to no confusion with this primary denotation.[1] All the capacities of substance are included under the general name *properties*. This term is also used with much lack of uniformity to include some or all of the powers.

Substances, Elements. — Different portions of substance exhibit more or less diverse groups of capacities. The

[1] By the denotation of a term is meant the object, idea, or concept, which the term replaces, that is, of which it is the name. By the connotation of a term is meant the attributes of that object, idea, concept, which are not included in the denotation but which are brought before the mind by the use of the term.

capacities of a designated portion are to some extent permanent, and therefore characteristic of that portion. Thus arises the suggestion that there are different substances, or substances of different kinds. The number of such different substances is very great, but each one differs from the one most like it by an abrupt and pronounced change in properties; in other words, the substances do not merge into one another by imperceptible gradations.

By physical and chemical processes, there can be separated or derived from all these substances a limited number, now about seventy, whose properties are of a notably simpler order than those of the majority. The most pronounced point of distinction between these elementary substances or *elements*, as they are called, and the others, lies in the fact that all known processes fail to separate them into simpler components. The elements have in common with each other and with all other substances certain properties, such as capacity for power through motion, capacity for gravitation, etc. The characteristic differences between them reside in the differences in their specific capacities for certain of the powers, notably for the chemical, magnetic, electrical, cohesive, elastic, and heat powers. The contrasts in these respects are great and seem at first sight random, but the properties of the various elements prove to be related by a very general though imperfectly apprehended law. Out of these elements all other substances are constituted, either through mechanical mixture or by chemical combination.

Those properties of the elements called capacities are permanent, if by this it be understood that under the same conditions a designated portion of a given element will exhibit the same capacities. By conditions are meant position and motion relatively to other bodies, and the action exerted by those bodies, or by their powers at the time. Laying aside consideration of the powers derived

from motion and from gravitation as being common to all substances, the quantities of other powers simultaneously present in a given portion of a given element vary from time to time, as already remarked. This is obvious enough as to heat, elastic power, and perhaps others. Some of the capacities also differ in the degree or intensity of their manifestation according to conditions, but these changes, however great, constitute a change of state or of condition of the substance, not a change of kind of substance. The contrast between the same substance in different states is so radical that regard to only a portion of the properties would lead to the supposition of separate substances; as for instance ice, water, and steam, or the solid, liquid, and gaseous states of any of the elements. The chemical properties, and the methods of chemical investigation, are the chief means of identification of substances. This arises from the relative constancy of these properties, and from the delicacy and simplicity of the methods. When chemical properties are under consideration to the exclusion of physical, the term *chemical substance* is sometimes employed.

No restriction or assumption has thus far been made or is to be made concerning the nature of capacities. It is possible, on the one hand, that the capacities are not resolvable into anything more fundamental. On the other hand, they may prove to arise from so simple a thing as a mode of motion of an ulterior substance of simpler properties. No proof of the latter view exists, and no assumption of its truth or probability will be adopted, but its possibility cannot be ignored in forming fundamental concepts.

Throughout physics, and especially at the outset, a term is needed which shall include as a part of its denotation all the properties called capacities. To this use no other word seems so well adapted, both by etymology and through the sanction of custom, as *substance;* the singular term being used in the abstract sense, and both singular and plural in

the concrete. The question of whether the powers are capable of existence separated from substance does not affect the concept of substance. As to this point, it may be here remarked that no one of these has been recognized except in association with some portion of substance possessing properties other than those related to this power alone. The case of radiation must be left to the paragraph on the ether.

Matter. — Influenced by the considerations just presented, but still more by others which can be offered only at a later stage of the general subject, we are accustomed to form a further concept underlying that of substance; namely, that of *matter*. Its presentation might more logically be deferred, but convenience in the development of certain contrasts as we proceed renders desirable the introduction of its definition here.

MATTER IS THE INERT CONSTITUENT OF SUBSTANCE.

By inert is meant intrinsically devoid of power of any kind. The more precise significance of inert must be left to the chapter to which the definition of matter strictly belongs. To that place may also be left the full discrimination between matter and substance. Briefly, however, we may point out that if the capacities of a substance prove ultimately not to be resolvable, then the concepts of matter and substance become coextensive. If on the other hand, they should prove to be resolvable into an effect or manifestation of "power" of some sort, or of a mode of motion, impressed upon an inert substance or carrier, then matter and substance would be distinct. The maintenance of this distinction in dealing with both experimental and theoretical physics will be found most helpful, particularly as tending to preserve that freedom from bias which is essential to sound scientific thought. While thus reasserting that we

have here to do with an open question, we may point to the theories of matter in Part II. as attesting the tenability of the view that the distinction is not a mere fancy or abstraction.

Mass. — All views of matter assign to it the possession of quantity. The term *mass* is used to denote this quantity, so that,

MASS IS QUANTITY OF MATTER.

The term is used to express either the abstract idea or the numerical quantity.

Weightal. — Quantity of substance as measured by "weighing" with the equal-arm balance will be denoted by the term *weightal*, which is discussed at pages 111 and 152. The term is inserted here merely for the reasons of convenience stated above respecting the term *matter*, and will not be employed in advance of its further explanation in a way to compromise the logical sequence of the main definitions.

CHAPTER II.

MOTION.

Definitions. — It will be necessary to employ certain terms respecting motion, and to avoid misapprehension these will now be defined. In the way in which they are here phrased they belong to kinematics, that is, to the mathematical study of pure motion as distinct from the physical study of the motion of material bodies.

Position. — The position of a point may be determined by its distance from at least three other points, or it may be expressed by its distance from a single reference point and the angles between a straight line drawn to it from the reference point and three other straight lines not all in the same plane. These angles fix the *direction* point from the reference point. Since there are no fixed lines or points in space the choice of all reference points and lines is arbitrary, and position is relative.

Motion is change of relative position. Motion is thus purely relative and the motion of a particle will appear different according to the reference point chosen. In describing motions we usually prefer to select for reference such points as appear to remain sensibly fixed with reference to each other during the time-interval dealt with.

Path. — The successive points through which a particle moves constitute its path. This may be straight, curved, or irregular to any degree.

The *Space* or *Distance* traversed by the particle is, unless otherwise specified, the distance passed over, measured along the path.

Direction. — If a particle is moved towards a reference point so that its successive positions lie along a straight line, then that line indicates the direction of motion or displacement of the particle. Motion is accounted positive when it is towards one end of the line, and negative or in the opposite direction when towards the other end. The particle is said to be moved or displaced along the line, or in the direction of the line, or towards the reference point, or in the direction of the reference point. In dealing with motion in space, it is customary to refer the direction of motion to an infinitely distant point unless otherwise specified. Thoughout a limited portion of space, all straight lines, whose direction is towards the same infinitely distant reference point, will be parallel, and will have the same direction. Throughout any such finite space, therefore, any particles moving along parallel straight lines will move in the same direction. Thus, in the general sense of the term, a particle is said to be moving in the direction of a specified straight line when it is moving either along that line *or along any line parallel to it.*

Still more generally a particle is said to be moving towards (or away from) a reference point, when its distance from that point is lessening (or increasing), however irregular or indirect may be the actual line of motion. In this case it is also said to be displaced towards (or from) the reference point. Or, if the direction is specified by means of a reference line, instead of the point, the particle is said to be moving in the direction of the line or to be undergoing displacement in that direction so long as its distance from a point infinitely distant along the line is continually lessening, whatever may be the actual course or path of the particle.

The direction of motion of a particle along its path is the direction of the path at the given time. If the path is not straight, this direction is that of its tangent at a specified point.

Velocity is the time-rate of motion (expressed, for instance, in feet per second or meters per second). If a particle moves over equal spaces in equal times, however long or short, its velocity is constant; if the spaces are successively larger, the velocity is increasing; if smaller, diminishing. The motion in these three cases is called respectively *uniform*, *accelerated*, and *retarded motion.*

The *Average Velocity* through any given time-interval is the space traversed in that time, divided by the time. If the motion is uniform, the average velocity is constant, however long or short the time-interval.

Instantaneous Velocity. — If the motion of the particle is varying, its velocity at any given instant is the limiting value of the average velocity during a short interval of time as this interval is taken smaller and smaller. If the motion is uniform, the instantaneous velocity at all times is, of course, identical with the average velocity. In general when the term *velocity* is used, instantaneous velocity is implied.

Rest. — This is merely a special case of motion in which the velocity relative to the chosen reference point is zero. Any particle will thus be at rest, whatever its motion relative to other points, when referred to a reference point possessed of the same motion relative to those points.

Acceleration is any increase of velocity. Acceleration may be continuous or sudden. Thus the velocity may be continuously increasing or may undergo increase for a brief time only, again becoming uniform; or may receive a succession of such impulses. Since all motion is relative, an increase of velocity differs in no respect whether the particle is initially at rest or in motion, so that the motion of a particle started from a state of rest by a sudden impulse and then continuing to move uniformly, is as much an acceleration as though the particle were originally moving at some uniform velocity.

Total Acceleration is the total increment of velocity in any stated time.

Rate of Acceleration is the time-rate of increase of velocity. The words *rate of* are usually omitted in speaking of this quantity. If this time-rate is constant, the motion is uniformly accelerated.

Retardation. — If the spaces traversed in successive, equal intervals of time are diminishing, this diminution of velocity is called retardation.

The **State of Motion,** *or simply the motion, of a particle is the instantaneous velocity and direction of motion referred to any chosen point and line.*

Change of State of Motion. — Any change of motion or of state of motion is therefore a change in either instantaneous velocity or direction or both, as thus referred. Thus, change of motion may be either acceleration, or retardation, or change of direction alone, or either acceleration or retardation combined with change of direction, depending upon the reference point chosen.

The state of motion of a particle at a stated point of its path would be shown by the motion of an imaginary point moving with the instantaneous velocity of the particle and in the straight line tangent to the path. The motion of a particle might be referred to a point moving in this manner beginning at any chosen instant. A little reflection will show that any change of motion whatever will appear, when thus referred, merely as an acceleration of the particle in some direction starting from a state of rest. *Any change of motion results therefore from the superposition of an acceleration as thus referred in a suitable direction upon some previous motion.* The acceleration may be continuous or an impulse.

Thus if a particle were moving uniformly in a straight line with regard to the chosen point, an acceleration in the direction opposed to this motion would produce retardation

or retarded motion. An acceleration in the direction of motion would produce accelerated motion. Acceleration at an angle to the direction of motion would produce change of direction and in general also change of velocity, but if in a direction always at right angles to the path, it would produce *only* change of direction.

Acceleration: Another Usage. — The term *acceleration* is often employed in abstract dynamics to include both retardation and change of direction as well as acceleration as above defined. In that usage positive and negative acceleration correspond respectively to acceleration and retardation as above defined. Following such usage all change of the state of motion would be called acceleration. This notation is discussed in Maxwell's "Matter and Motion" [London and New York, 1882, 18mo, p. 128] which should be read by students of this subject.

The terms *kind* or *mode of motion* are to be distinguished from state of motion. They refer to the motion which a particle, or system of particles, has been undergoing or will undergo; that is, they refer to the past and future motion of a particle and not simply to its instantaneous motion. Thus the motion of a particle in a circle would be a kind or mode of motion, not a state of motion.

Two modes of motion to which reference will be made are translatory and rotary motion. A particle, or a body of sensible size, is in translatory motion when the paths of all portions of it are parallel straight lines. A particle, or a body of sensible size, is in rotary motion when each point of it is moving in a circular path about some imaginary axis. The two motions may be, and usually are, combined in the motion of any actual object.

Free Motion. — This term belongs to dynamics and not to kinematics. It is the motion of a body when not restrained in any direction, that is, when not interfered with by the power of another body (p. 3).

A body may be free to move in any direction, even when subject to any number of external actions, provided the effects of these actions balance one another and remain balanced in whatever direction the body may be moved. The conditions necessary for perfectly free motion are difficult to fulfil in practice, but nearly perfect freedom along a line or in a plane may readily be obtained. Thus a ball sliding upon smooth ice has nearly perfect freedom of motion in a horizontal plane; a billiard ball has some such freedom upon the table; a ball upon the bowling alley is also nearly free in a horizontal plane, or a carriage rolling upon a smooth level floor.

CHAPTER III.

ENERGY.

Laws of Motion. — The most casual observation shows us that, under ordinary conditions, bodies set in motion come to rest sooner or later, unless some effort is exerted to maintain their motion. To ascribe to motion a tendency to decay is therefore most natural, and this was long done. Little further observation however is required to show that the rate at which the motion of any given body diminishes, depends upon the conditions among which the body moves. Thus, if started each time with the same velocity, a ball would move but a short distance over a pebble-covered road, further over a good road, much further still over a smooth floor or over ice, and still further if free in air. It is obvious that the irregularities which constitute the roughness of these successive surfaces oppose the motion of the ball. Also, opposition is afforded by the air, as we know from moving the hand rapidly through it. In these cases, then, the less the opposition, the longer the motion persists. By the multiplication of such evidence, confirmed by studies of the motion of celestial bodies, and by general experience, we arrive at Newton's first law of motion.

Using the terms with the meanings already assigned to them, this law may be expressed thus: —

The state of motion of any body remains constant, except in so far as changed by external action.

It is to be noted that this law is not a direct statement of observed fact, but an inference as to what the motion of a free body would be, based on the observed motions of bodies

which are not free. No actual body is found which is entirely free.

A more direct general statement of the observed facts may be put in the form that: —

All observed changes in the state of motion of bodies are due to discoverable external action.

Throughout these statements the body considered is assumed to be permanent as to the relative position of its parts. It is easy to show later, that what is here referred to as the motion of the body, is the motion of a point occupying a definite position relatively to all portions of the body, and that the above law applies to the motion of that point whether the parts of the body be in motion relatively to each other or not.

Inertia. — The doctrine or principle, less correctly called the property, of inertia, is implied in Newton's first law of motion. It may be more explicitly stated thus: —

No body has power to change its own state of motion.

The doctrine or property of inertia is a purely negative one, a mere denial of the possession of a certain power by bodies. It does not ascribe to bodies or to substances any property in virtue of which external action is required to change their state of motion, or in virtue of which they possess when in motion any power relatively to other bodies; although such a character is often ascribed to this property. It is of course understood that this expression does not deny to a body the power of relative change of position of its parts. It refers merely to the relation of the body as a whole to external objects.

Definition of Energy. — We now arrive at the most conspicuous and important of modern physical concepts; that of *Energy*.

By Newton's first law of motion, just stated, any change in the state of motion of any body, that is, any deviation from uniform velocity of motion in a straight line, is due to exter-

nal action. To the *power to exert* this action, is given, in all physical science, the name, *Energy*. We may define the term thus: —

ENERGY IS POWER TO CHANGE THE STATE OF MOTION OF A BODY.

The motion is to be referred to any point not involved in the change of motion to be observed. This is the most general mode of reference possible, and therefore the best for the purpose of the definition.

Energy stands to change of state of motion as cause to effect. Without energy there can be no change in state of motion; and consequently, wherever change in state of motion is in progress, there energy must be in action. Such change affords indisputable and sufficient evidence of the presence of energy in any specific case.

The form of this definition is a departure from prevailing usage, as shown in Chapter XII., with the reasons therefore. Its merits or defects must be developed through its application to all physical phenomena and knowledge, and test of its value will thus be afforded at nearly every subsequent page of this book. Should it fail to discriminate correctly in every case between that which is energy, and that which is not, as known by other reliable tests (if such there are), it would thereby be condemned. If it be found defective in logic, it must be abandoned. If it be found inconvenient or difficult of comprehension, then it should be replaced by a better.

The definition may, at first sight, appear insufficient, as energy manifests itself to us in such a multitude of diverse ways, most of which we do not recognize as due to motion at all, and many of which are not known to arise from motion. Thus, energy produces in us the sensations of light, color, taste, smell, sound, warmth, and so on. It gives rise to all the phenomena of Nature around us. Still, we are un-

able to trace the nature of the energy in most of these occurrences to even the limited extent of ascertaining whether it is or is not due to motion of some sort. In a few classes of cases only do we know that it is so. But we do know that energy, in whatever form it manifests itself, can be made to produce change in state of motion of bodies. In other words, it can be changed into energy of motion of a material body. Proofs, or rather illustrations of this, will be given in the following chapter. Further, energy of motion is the sole form of energy of whose nature we have adequate knowledge. Again, the definition arrived at through energy of motion, and advanced above, appears to be more simple, and more easily grasped by the beginner, than those usually employed. It also renders possible the introduction of the subject of energy at the very outset of the study of physics with entire logical consistency. These and other considerations, some of which appear elsewhere in due course, together with its ascertained effectiveness in developing the other concepts of physics, have decided the author in its adoption. Without further present comment on this definition, we shall proceed at once to its applications.

In dealing with the motion of actual bodies with reference to energy, the selection of the reference point for the motion is often of considerable importance. In the foregoing general definition the motion is referred to any point whatever. In any particular case we are concerned with the energy of motion of only a limited number of bodies. Such a set of bodies may be called a system. In any such system we are ordinarily concerned with the motion of one or more bodies relatively to the system as a whole. The state of motion of these bodies would then be their motion relatively to the system as a whole, or relatively to some point at rest with respect to the system, that is, possessing relatively to any other body the same motion as the whole system. Some little care is frequently necessary in the selection of such a reference

point. In dealing with objects near the earth's surface, this forms a convenient reference point since such systems of bodies would generally be at rest with reference to this surface, or would possess some known motion with regard to it. In a system in motion relatively to the earth's surface, we frequently select some member of the system which for sufficient reasons we know to be not involved in the changes of motion with which we have to deal. The most general of all physical reference points and directions are of course those of the fixed stars.

Muscular Energy. — As far as concerns the continuity of the set of physical definitions presented in this book, all that is given under this heading, "Muscular Energy," might be omitted. The purpose in retaining it is merely to indicate an easy transition from ideas familiar to almost every one to a more abstract and purely scientific definition. As will be again noted, however, this definition is in no wise dependent upon, or restricted by, this introduction.

Whatever point be chosen for reference, the motion of bodies is in general undergoing change. If now by our own action we produce in moving bodies or in those at rest, any change of motion which they would not otherwise undergo, we must exert muscular effort. That we must do so follows directly from the first law of motion (p. 18). We also become aware of it through direct experience. For the *power or capacity to exert such muscular effort* we have a well-recognized term, *muscular energy,* or *energy.*

Exercise of the mind or nervous system accompanies muscular effort, but is directive of, rather than a part of, the muscular energy.

The term *muscular energy* or energy as thus defined conveys to every one an entirely familiar and sufficiently definite conception. This and the further fact that energy is used in much the same sense with reference to inanimate objects in physics, renders this conception a quite suitable starting-

point from which to approach the physical concept. The purely physical definition is in no way based upon or derived from the definition of muscular energy, so that any want of precision in the latter does not vitiate the scientific definition. The notion of muscular energy, however, gives concreteness or tangibility to the otherwise abstract idea, — a consideration of the utmost importance in the elementary presentation of the subject. It may here be noted, however, that although the sensation accompanying muscular effort is in no way directly involved in the physical definition, yet it forms an ineradicable part of the connotation (foot-note, p. 7) of the term, and exerts a very considerable, perhaps not altogether wholesome, influence on certain other physical concepts, as will be later indicated.

We have been dealing with the general case of the motion of *any* body, and therefore with the motion of a body not entirely free. The effort producing change of motion of such a body is expended in two ways. One portion goes to move the object against the obstruction to its free motion. The other part is exerted in merely accelerating or otherwise changing the state of free motion of the object.

That muscular effort — and hence energy — is necessary in changing the motion of a free body not only follows from the converse of Newton's first law, but is easily illustrated by familiar experience. For instance, to throw a ball horizontally, that is to set it in rapid motion, requires a considerable effort. To maintain the speed of the ball, however, obviously calls for very little effort, since it will proceed a long distance with no application of effort whatever from without. The effort of throwing the ball is mainly expended in the mere acceleration of its motion. A large door swinging on well-oiled hinges can be easily kept moving at any desired speed after that velocity has been reached, but to bring its speed up to that point requires a distinctly greater effort. To roll a carriage over a smooth level floor,

a car upon smooth level rails, a loaded sled on ice, all afford examples of effort expended in mere acceleration of bodies. In all of these it is easily perceivable not only that effort is expended in acceleration but that the amount of effort depends upon the rate of acceleration, that is upon the time in which a definite increase of velocity is produced, the effort being measured by the push or pull exerted — terms whose ordinary meaning is sufficiently definite for the present purpose.

The introduction of the term *energy*, by means of its familiar association with muscular effort, might usefully precede the scientific definition of the term in the elementary presentation of the subject.

CHAPTER IV.

FORMS OF ENERGY.

Recognition of Forms of Energy. — Having thus postulated a definition of energy, we proceed to ascertain in what forms energy presents itself. The definition affords us a test for energy. Wherever change in state of motion occurs, energy must be in action, whether or not we are able to determine in just what the energy consists or precisely where it resides in the given instance. If the definition is adequate, this test will never lead to the mistaking of anything else for energy, or ultimately to the overlooking of any form of energy.

The disclosure of all the ways or forms in which energy manifests itself demands ultimately the examination of all the phenomena of the universe. Happily, however, this infinite task is reduced to one of limited magnitude through the classification of phenomena into a few great groups such as the phenomena of cohesion, of gravitation, of elasticity, etc. Each of these groups then corresponds to a property (of the kind which is here called a "power") of bodies or substances. And as we apply to each the test for energy, we shall find that each is due to some manifestation, or as it is called some *form of energy.* How far these apparently different forms are in their real nature identical we must subsequently discuss. We thus arrive at a system of nomenclature for the forms of energy which, however empirical, is so interwoven with colloquial usage and with the historical terminology of physics that it must long survive.

Kinetic Energy. — It is an observation of the most familiar

kind that a body in motion relatively to another possesses power to change the state of the other's motion by collision. The first body therefore possesses energy with reference to the second. It is also easy to discover that the amount of the power possessed by the body depends upon its velocity relatively to the second. But in ordinary cases of collision, so many incidental actions of different kinds occur that the conditions must be much simplified to demonstrate the law of this energy.

Take two ivory balls A and B, of equal diameters and as nearly as possible alike in all respects. Suspend each by a long silk thread, or better by two threads diverging upward. Place each ball so that when hanging free the two just touch each other, and so that their point of contact is in the line of free swing of either. The two balls will then have nearly perfect freedom of motion when moving in a horizontal plane through their point of contact, or at least in one direction in that plane. Let B remain at rest, and draw A to one side and release it. Just before reaching B, it will have acquired a definite velocity. On colliding with B, A will come completely to rest (or sensibly so), and B will move off with a considerable velocity. The motion of B has thus undergone acceleration. By the definition then, energy has been exerted upon it. In what did that energy consist?

By close scrutiny of the conditions of the experiment we can discover, beyond the changes of velocity of A and B, no other changes except certain minor ones. The latter consist chiefly of vibrations of the parts of the two balls and of the surrounding air, and of a minute amount of heat developed in the balls. These minor changes may be disregarded at present as unimportant. Subsequent investigation of them would show that due allowance for them only strengthens the inference here drawn. During the interval occupied by the collision, the only sensible change was the loss of velocity by A and the acquisition of velocity by B.

A then must have possessed energy with reference to B. Also it must have possessed this energy in virtue, partly at least, of its velocity with reference to B.

If A is given a higher velocity, the acceleration of B will be greater; so that the energy of A appears to increase when the velocity is increased.

If a larger ball of the same substance be used instead of A and be given the same velocity as A, it will impart to B a different acceleration. If balls of different substances be used, differences in the acceleration produced will be observed. But most substances are imperfectly elastic so that in the collision of such bodies the incidental changes, such as crushing, production of heat, etc., are not negligible.

From the foregoing, however, we are justified in concluding that a body in motion possesses energy in virtue jointly of that motion, and of some property of the body itself. This form of energy is called *energy of motion* or *kinetic energy.*

The foregoing inference is confirmed by universal experience with bodies and substances of every sort. The law of kinetic energy will be demonstrated in a later chapter, and a more general method of illustration will be described.

Kinetic Energy of Rotation. — If a body be in rotation about a definite axis, each point or particle of the body has at any given instant a definite direction and velocity of motion. The direction is that of the tangent to the path which it is describing, and its velocity is the instantaneous velocity. The body may therefore be regarded as made up of an indefinite number of very small portions, each of which has its own velocity and therefore its own amount of kinetic energy. The sum of all these amounts will be the total kinetic energy of rotation of the body.

Molar, Molecular, Atomic Kinetic Energy. — A distinction is convenient between the kinetic energy possessed by a single molecule, and that possessed by an aggregate of

molecules, — admitting for the moment the molecular hypothesis of the constitution of bodies. The energy possessed by a single molecule in virtue of its velocity is called *molecular kinetic energy.* That possessed by a body composed of more than one molecule is called *molar kinetic energy,* and similarly we may have *atomic kinetic energy.* These may be due either to motion of translation or to rotation.

Gravitation Energy. — One of the most familiar phenomena exhibited by bodies may be stated in this way: Every body tends to approach every other body. In other words, any two bodies *will* approach each other unless prevented from so doing by some external means.

This at first sight appears directly contradictory to the doctrine of inertia, but if we adopt the views above presented with regard to inertia and energy, and pursue the line of inquiry just enunciated, this contradiction at once disappears. If analyzed, the above statement will be found to mean that each of any two bodies, if allowed entire freedom of motion, has its motion accelerated relatively to its previous state of motion; that is, the state of motion of the body is changed. Therefore, according to the criterion which we have imposed for energy, there must be energy of some sort in action, causing the change of the motion of these bodies. Here then we have directly an evidence of the existence of energy.

What is the character of this energy which causes gravitation? To this no satisfactory answer can be given, but certain attempted explanations are presented in a later section. There is no discoverable change in the bodies concerned, or in any of their surroundings, which takes place at the same time and in the same measure as the change of motion of the bodies. There is no direct evidence that any body or bodies whatever are producing this change through collision with the bodies concerned;

that is, there is no evidence that this action of gravitation is due to any kinetic energy. Inspection fails to show whether the energy resides upon the bodies or is external to them, and has failed also, thus far, to give a thoroughly satisfactory hypothesis as to its nature. We are, however, in no way relieved by this fact from the inference that this energy exists; and this conclusion leads us into no contradictory positions, but on the other hand aids in establishing a simple and satisfactory system of ideas.

We thus have demonstrated the existence of energy associated with substances in a way which is different from kinetic energy, or at least which we are unable as yet to demonstrate to be due to kinetic energy. We are therefore obliged to content ourselves with saying that this is *another form of energy.*

Briefly, then, our second form of energy discovered, is that which causes the phenomena of gravitation.

Heat. — We know familiarly that through various machinery, such as a steam engine, etc., we are able to set objects in motion, and we know that the power to produce this result is derived from the heat which is generated under the boiler of the steam engine. We have, then, another case in which the motion of a body is changed, and in this case we are able to go a step farther than in gravitation, inasmuch as we can say that this energy is identical with something with which we are familiar in other ways, namely, heat, — whatever the mechanical nature of heat may prove to be. The above illustration is, of course, not a rigid proof; but a perfectly strict demonstration that heat is convertible into molar kinetic energy is easily possible.

Heat, then, is still another form of energy; although the nature of heat energy is a matter of hypothesis. If we admit the molecular constitution of bodies, then it is easily shown that heat is probably due to the vibratory motion of the molecules within a body, and that it is in reality the

total kinetic energy of vibration of the molecules apart from any translatory energy of the body as a whole.

Energy of Elasticity. — If a bent spring be released, the substance of the spring is set in motion, that is, its state of motion is changed. Or if the spring, when released, be allowed to act upon a body separate from itself, the motion of that body will be accelerated. Some form of energy must, therefore, be here in action. The change which we recognize as taking place concomitantly with this exhibition of energy is a change of form of the spring. Some portions of its substance are extended beyond their normal length while others are compressed. This change of form cannot be the energy, but merely indicates that the energy is brought into action through change of relative position of parts of the substance of the spring. Here again we are entirely ignorant as to the mechanism of the form of energy. This property of bodies, which is called elasticity, is somewhat complex in its manifestations and possibly in its nature; but without attempting to analyze the phenomena thoroughly, we are evidently at liberty now to say that wherever the property of elasticity is in action, a form of energy is present. This form may be called the energy of elasticity, or may be given the not very euphonious name, elastic energy. From the fact that this energy is always brought into action through the deformation of the body, we might infer that the energy was resident upon the body; but the most that can be positively affirmed, in the case of solids and liquids at least, is that it is intimately associated with them. A fundamental theory of the nature of elasticity is stated later, in the section on the *Vortex-atom Theory.* A theory explaining the molar elasticity of gases is stated under the *Kinetic Theory of Gases.*

Cohesion Energy. — When we attempt to pull apart any two portions of a solid body, we find that the body offers resistance, and that the pull must exceed a definite amount

to break the object. Again a small portion of any liquid, if free from external disturbances, will assume a spherical form. A rain-drop is nearly spherical. A drop of water or mercury upon a horizontal surface which it does not wet assumes the shape of a flattened sphere, the flattening being due to the interference of gravitation. These phenomena and others analogous to them indicate that the particles of a body, when in close proximity, tend to approach one another, and will do so unless prevented by some external action.

We know, from commonest experience, that the size of bodies is due in part to the presence of heat in these bodies, and that if some heat is withdrawn the bodies become smaller, that is, their particles approach one another. This approach is due to the property of cohesion and is a direct evidence of the action of some form of energy, since it involves a change in state of motion of the parts. The motion of the parts of the drop of liquid in assuming the spherical form under the action of cohesion is similarly direct evidence of the action of energy. There is present, therefore, some form of energy which causes the phenomena grouped under the name cohesion. The phenomena of elasticity when bodies are stretched, and those of cohesion, are closely interwoven. But no embarrassment is introduced by assuming that, distinct from energy of elasticity, there is a form of energy which may be called the energy of cohesion. This form appears at first sight to resemble that of gravitation, but its effect appears to vary with the increase of distance between the particles much more rapidly than the law of gravitation indicates. Certain explanations of this apparent discrepancy, however, have given plausibility to the view that cohesion is identical with gravitation (cf. Part II.).

Chemical Energy. — In general, it is found that portions of any two different elements show a tendency to combine

with each other, forming a compound with properties different from those of its constituents. These compounds in turn manifest certain tendencies towards combination or other chemical reaction with each other. The process of combination is attended, in very many instances, with the evolution of heat or with change of volume. The production of heat in such a case where no external condition is changed, may be regarded as evidence that the chemical action was the result of the action of energy in some form; for this heat may be transformed into molar kinetic energy, and thus the chemical action is made to give evidence that it is an action of energy. The fact that this process consists of two steps, first, a production of heat; second, a transformation of heat into molar energy, is not to be regarded as impairing the validity of the inference. For our criterion for determining what energy is, does not imply that the change of state of motion must be produced directly. Indeed if we analyze the very first instance which we have stated, namely, acceleration of one elastic body by impact of another, we find that this operation also is not a direct one. The energy possessed by the moving body is entirely transformed into energy of elasticity, and again retransformed into kinetic energy, during the impact.

In case of the change of volume during chemical combination, where this has not been brought about by any external influence, this change is direct evidence of energy, under our criterion.

There are many cases of chemical reaction where the absorption of heat, or of energy of some other form, from without attends the operation. This energy disappears, that is, no longer continues to exist in its original form, but it may be traced, on further analysis, in the physical or chemical changes which occur during the chemical action. Without pursuing this into detail, it is clear that we are justified in assuming that chemical action and chemical af-

finity, so called, are due to a form of energy which we will call chemical energy.

A more vivid illustration of the existence of chemical energy is in the action of explosives. Here the chemical energy gives rise not only to violent change of motion but to other actions by which we may recognize energy.

With the nature of this form of energy we are as little acquainted as with that of gravitation or elasticity. But a very remarkable stride in the direction of explanation has been made under the *Vortex-atom Theory*, as later described.

Electrical Energy. — Under certain conditions a conducting body, such as a metallic wire, exhibits properties which we ascribe to a current of electricity flowing through the wire. The conductor containing the current becomes heated by the passage of the current, and by proper disposition of other conductors about this one, motion may be set up, as in electric motors and various other well-known forms of electrical apparatus. These actions begin and end with the current, and are clearly due either to it or to the same cause which produces it, and by our criterion must be due to a form of energy. This we call electrical energy.

This form of energy is manifested in another way. Under certain well-known conditions, conductors of electricity when insulated from other conductors become charged, as it is called, with electricity. Two such conductors tend to approach or recede from one another according to the character of the charges upon them, that is, they will so move unless prevented; therefore these charges are either due to or attended by a form of energy which appears to be fundamentally identical with that which attends currents of electricity. The difference in the phenomena exhibited by electricity at rest upon charged bodies, and electricity in motion in conductors, appears to be due solely to the motion. We do not, for our present purpose, need to examine further the many properties and

phenomena of electrified bodies, since those noted are sufficient to establish the existence of electrical energy.

Magnetic Energy. — The natural lodestone is capable of moving a small mass of iron. The artificial magnet has the same power. This power results from a condition apparently attending and always associated with portions of iron. It is also possessed in widely varying degrees by nickel and some other substances and is called magnetism. The closely related property known as diamagnetism is possessed by a large number of substances. This power to move bodies arises, therefore, from some form of energy closely associated with the magnetic body. It may be called magnetic energy. Of its nature we know, first, that it is very closely related in some way to electrical energy; second, that at present the most satisfactory explanation of its action is upon the *assumption* that magnetic energy and electrical energy are identical.

Energy of Sound Vibrations. — We know that a body transmitting sound is in a state of vibration. Its particles are moving to and fro in a systematic manner, each particle passing from a state of maximum velocity to a state of zero velocity and back again at regular periods, the motion being in the direction in which the sound is propagated. When in this condition of maximum velocity, each particle possesses a definite maximum of kinetic energy. As it passes in its vibration towards its state of zero velocity, it gradually yields this kinetic energy, which is transformed into energy of elasticity of the body. This energy of elasticity is in turn expended in accelerating the motion of the particle, and the process is thus one of successive transformation of kinetic into elastic energy and the reverse. The result is a production of a wave of energy through the transmitting body. The total of all this kinetic and elastic energy within the body may be called the energy of sound contained by that body. It is not neces-

sary to choose between the subjective and objective definitions of sound, but for clearness we may say that the sensation of sound originates in, and is transmitted by, the vibratory energy of elastic substances, and that this vibratory energy consists, at any given instant, partly (equally) of kinetic and elastic energy in the substance. This resolution into kinetic and elastic energy is capable of rigid proof, so that the energy of sound is not to be classed as a separate form.

There are other known forms of vibration energy, such as that of the swinging pendulum, of the planets in more or less elliptical orbits, etc. Each of these consists, however, of a joint action of kinetic energy and gravitation energy or some other form. They, like sound energy, are therefore not to be classed as new forms.

Radiant Energy: Light, Radiant Heat, Electro-magnetic Radiation. — It is a matter of observation that hot bodies produce upon other objects in the vicinity certain effects, even when entirely unconnected with them by any other substance than that which can exist in the most perfect vacuum attainable. The nature of the effect depends upon the receiving object. Thus if the body be sufficiently hot, the human eye is affected in such a way as to produce the sensation which we call light. The blackened bulb of a thermometer is so affected that the thermometer indicates a rise of temperature, that is, indicates that heat has been introduced into the bulb. If the receiving object is a sensitive photographic plate, chemical action is promoted. Of these three phenomena, one, namely the production of heat in the bulb of the thermometer, affords evidence that by whatever process the hot body may have produced its action upon the thermometer, the essential part of the process consisted in the transmission of energy of some form to the thermometer; for heat was produced in the bulb of the instrument. Now this heat can be made to accelerate the

motion of a body; therefore that which produced the heat must have been a form of energy. This power of a hot body is called radiation. Radiation, therefore, in this case is a process of transmitting energy in some way to objects not in direct contact with the radiating body. Hypothesis is necessary to explain the nature of this process of transference, but we need not enter into this explanation at the present point since the only thing which we wish here to establish is the existence of a form of energy by which the transference is made; and that has already been done.

This power of radiation is possessed undoubtedly, not only by hot bodies, but by all bodies with temperatures above the absolute zero; also by bodies in which rapid oscillations, or rapid periodic changes of position, of electrical charges, are taking place; also by bodies in which rapid magnetic changes are occurring; also by certain vacuum tubes through which high-pressure electrical discharges are taking place. The effects produced in these various cases are not all the same, but they are all equally capable of being proved to be due to a form of energy transmitted from the radiating body to the receiving body. This is called radiant energy, or at least this term may for the present be held to cover it. It is supposed to be due to the vibrations of the (hypothetical) luminiferous ether which is supposed to be universally distributed through space.

Muscular Energy.—In a preliminary classification, it would be necessary to include this title. But investigation shows that the energy of the muscles is apparently not an independent form, but only a more or less complex manifestation of some of the foregoing forms; namely, cohesion, elasticity, and perhaps others, with chemical energy as a precursor or concomitant. The exercise of muscular energy is in general controlled by the brain and other nerve centers. But here again there appears to be no introduction of energy of other than the foregoing

forms. The term *vital energy* is also in somewhat general use in a non-scientific way, to denote the energy of living things. There appears to be no evidence that there is here another form. Rather, the term seems a convenient one to include the sundry known forms which, in association, produce the phenomena characteristic of animate things. A condition certainly exists in animate objects, in virtue of which energy-changes occur which would not occur otherwise. But to what this condition is due, remains to be discovered. The experimental difficulties, attendant upon investigations of energy-changes in living organisms, have prevented progress in that direction, so that perhaps the most that can at present be asserted is that there is no evidence of any form of energy peculiar to living things. Hence muscular and vital energy are not terms denoting independent forms of energy.

List of Forms of Energy. — We have thus demonstrated the existence of at least nine forms of energy which, so far as we have yet proved, are distinct from one another; these are: —

1. Kinetic Energy,
2. Gravitation Energy,
3. Heat,
4. Energy of Elasticity,
5. Cohesion Energy,
6. Chemical Energy,
7. Electrical Energy,
8. Magnetic Energy,
9. Radiant Energy.

This list includes all known separate forms.

To determine how far these nine forms are mutually independent, demands a knowledge of the nature of the various forms. This knowledge, science does not now fully possess. Theories there are, but of direct knowledge, next to nothing. These theories indicate that heat is molecular kinetic energy; that cohesion and gravitation energy are identical; that radiant energy is a form of vibration energy (of a substance called the ether); but these theories are not

a sufficient warrant for simplifying the classification. Hypothesis goes even further and suggests that each form arises from a mode of motion of some portion or portions of substance or of matter, and that therefore all energy is kinetic. This proposition will be recurred to, but however alluring it may seem, the fact that it rests only on hypothesis must not for an instant be overlooked. It is often accepted as a personal belief, but beliefs frequently rest on even more slender foundations than does this one, and are not to be propounded as scientific facts.

No assumption will be here employed to the effect that all energy either is or is not kinetic in its nature. It may, however, be said that in the present state of our knowledge, while it would be distinctly unsafe to *assert* that all energy is kinetic, it would not be unsafe to assume as a working hypothesis that it *may be.* To many minds, such a hypothesis would remove a sense of mystery or vagueness, and to most persons perhaps it not only would prove a decided help, but, if always held in leash as a hypothesis, it would lead into no serious misapprehension.

Classification of Energy. — The above classification of forms of energy is based upon the properties which substances exhibit. This seems to be a natural mode of classification. A decidedly different method has been recently advocated by some German writers. It may be here remarked, however, that as the term *form* has been so long appropriated in the ordinary classification based on properties of bodies, it would seem to have been wiser to select another term by which to designate the classes under this new system. Mention may here be made of the division of all energy into two groups, kinetic and potential. This will be considered in another section.

Mechanical Energy. — Those forms of energy which play a part in the operation of machinery are roughly grouped in a class called mechanical energy. This is not a definite

or important classification, but one which is more or less familiar. Perhaps it may properly be said to include other forms than heat, chemical, electrical, and magnetic energy, that is, any other than these four forms might be referred to as mechanical energy, although radiant energy would hardly be so classified.

CHAPTER V.

FORCE.

Definition of Force. — The state of motion of a body at a given time is fully determined by the velocity and direction of the motion at the given instant. The motion is to be referred to any point not involved in the change of motion to be observed. This is the most general mode of reference possible, and therefore the most convenient. The definition of energy (p. 20) states without qualification that the *power* to change the state of motion of a body is *energy*, and it therefore follows that nothing other than energy can produce that change.

Energy may exist without *producing*, or even without *tending* to produce change of motion; *e.g.* the kinetic energy of a freely moving body. When, however, change of motion is taking place, energy in some form must be acting upon the body to produce that change. Energy may be in action against a body without changing the motion, owing to some restraint upon the body's freedom of motion. In that case the energy will be producing merely a tendency to change of motion, by which we mean that change *will* be produced if the restraint is removed. This tendency to change may exist not only when the body is at rest but also when it is in motion of any sort. Thus energy may be producing a tendency to change of motion in a body, or portion of substance, whether free, or constrained, and in any state of motion whatever; rest being a special case of motion. This action

of energy is *force*, and force can be produced by nothing but energy. We may, therefore, define force thus: —

FORCE IS THAT ACTION OF ENERGY BY WHICH IT PRODUCES TENDENCY TO CHANGE IN STATE OF MOTION OF BODIES.

The relation between this definition and that of Newton and others will be considered in Chapter XII.

The relation and distinction between force and energy expressed in this definition cannot be too strongly emphasized. Energy is the agent, — the doer, — the cause, — that which performs the action and produces the effect. The effect produced is "tendency to change in state of motion," which is a condition thus imposed on the body. The operation, process, effort, mode of acting, *action*, by which energy produces this effect is force, as here defined. Force is thus that action by which the cause — energy — produces a particular effect; namely, tendency to change of state of motion. And action is used in its generally recognized sense of operation, etc., as just stated, not in the sense of effect.

The operation of imposing upon or maintaining against a body the condition denoted by "tendency to change in state of motion," in other words, the exertion of force, demands no expenditure of energy by the acting source of energy. For example, a compressed spring expends none of its energy of elasticity, however long it continues to exert force (pressure) against the objects which are preventing its recoil. The moment, however, that any portion of the restraint is removed, an action of the elastic energy occurs which is not force but which does involve the expenditure of some of that energy, — a process to be described later under the name of *work*. To fix still more clearly the limit of the meaning of force, we may anticipate slightly. Whenever a given source of energy is producing upon a given body tendency to change in state of motion, it is by definition exerting force against the body, whether or not the

body is in motion of any sort whatever. The mere exertion of the force withdraws none of the energy; and the force, being the action, continues only so long as the action endures. If the body has partial or complete freedom of motion in the direction of the force (the direction in which the body would move if entirely free), then the action of the energy will be twofold; the energy will both exert force and "perform work." The latter process consists in the transference of energy from the acting source to the body acted upon, as will be duly shown. Both force and work are thus names of operations or processes, and do not denote forms or effects of energy. Neither of them is an entity, or quantity having permanent existence, in the sense in which energy and matter are often called entities. Force can produce nothing in the sense in which a cause is said to produce an effect. Energy is the cause which, through the action called force, produces the effects so often erroneously ascribed to force; namely, the various changes in state of motion of bodies.

It is a most important result of general observation that, when energy is exerting force, two or more bodies are always involved in the action. That is, wherever one object is observed to possess a tendency to acceleration in a certain direction, another object is always discoverable (directly or indirectly) towards which or away from which the first tends to move, and which also has a tendency to motion towards or away from the first object. Often more than two objects are involved in the same action upon another. Familiar examples are as follows. In the collision of two elastic balls, the elastic energy of each exerts an equal force upon the other. Gravitation energy exerts a force tending to move the planet towards the sun, but also exerts an equal and opposite force tending to make the sun move towards the planet. A body near the earth's surface is urged by gravitation energy towards the earth, but the earth is urged

by gravitation energy with an equal force towards the body. Wherever change of state of motion of one body with reference to another is in progress, force is being exerted by energy upon one or the other of the two bodies, but not necessarily upon both. From the consideration of the mere motion of these two bodies, it is not possible to determine upon which the energy is exerting the force. But by scrutiny of the surroundings or by extended investigation, it is generally possible to arrive at a correct decision on this point. It is, however, evident that in endeavoring to ascertain the precise locality and amount of the real forces involved in any action, a choice of reference points for the motion is of the utmost importance. Mere acceleration affords knowledge and measure in many cases of only a portion of the force, either by reason of the choice of reference point or by reason of constraint in the motion of the bodies.

How Energy produces Force. — The actual manner in which energy produces the action which we call force must obviously depend upon the nature of energy itself, and of the particular form of energy exerting the force. Upon this point very little can yet be discovered through direct observation. We know that kinetic energy is transferred and force exerted in the collision of bodies. But we also know that the transference is through the energy of elasticity, and that it is elastic energy which exerts the force between the two bodies. Of the intimate nature, or mechanism, of elastic energy nothing is directly known. We have only *hypothesis* to assist us here; and the *Vortex-atom Hypothesis* described later is the only one which affords an ultimate explanation of elasticity by accounting for it through a peculiar mode of motion. The various atomic hypotheses explain the elasticity of bodies by assuming perfectly elastic molecules, out of which the bodies are supposed to be constructed. They fail to account for the elasticity of

the molecules. An examination of the statement of these theories given in Part II. will show how they account for certain forces as consisting of a bombardment by elastic molecules. This idea is a helpful one, but it must always be remembered that it is merely a hypothesis. The student of physics cannot be too careful in discriminating between such hypotheses as the molecular theories or deductions from them, and such fundamental facts as the laws of energy and of force as an action of energy.

Recognition of Forces. — We become conscious of the action of some forms of energy through our senses. Thus we are conscious of the force called weight when we hold a heavy object in the hand; and in various ways we recognize pressures which are exerted upon us by external objects. This consciousness does not necessarily imply what may be termed a sense of force; all that is here meant is that we are conscious of a muscular effort which is necessary to resist an external force, or to exert a corresponding force upon surrounding objects. We can gain through sensation a rough measure of the amount or quantity of force exerted upon or by ourselves. The purely physical methods, however, for the recognition of force are two. First, by the change in state of motion of bodies; second, by the distortion of bodies.

From our definition of force, it is obvious that when we can see a change in the state of motion of one body with reference to another, we know that force is being exerted upon one or both of them. If, however, either or both of the bodies are under recognized restraint relatively to each other, and we find that on removing that restraint a change of motion occurs whenever and as often as the restraint is removed, then we infer that force is being exerted upon one or both during the time of restraint.

If a spring be bent, or if any elastic body be strained out of shape, it tends to regain its former unrestrained shape or

position. If when so strained it be set free, it will return to its normal form unless it has been strained beyond the limits of perfect elasticity. In so doing it will change the state of motion of its own parts and of other bodies connected with it. It is, therefore, exerting force when under strain. This is universally true of all solid bodies; so that whenever we observe deformation to exist in any solid, we know that the body must be exerting elastic force. This furnishes us with our second physical means of detecting force.

Resistance. — Whenever a body is not free to move in respect to any energy acting upon it, we are accustomed to say that there is some "resistance" opposing its freedom. If we inquire into the nature of this resistance, we shall see that it is always due to the action of some form of energy, the action being of such a nature as to constitute a force opposing the body's motion.

The sufficient evidence that all resistance is due to the action of energy lies in the fact that through resistance change in state of motion of bodies occurs.

That which we name resistance, however, is not the energy, but merely that action of it which produces the *tendency* to change in state of motion. Therefore, by our definitions: —

All resistance is force.

In raising a body from the earth, we encounter a resistance. This we know as weight, a force due to the energy of gravitation. In moving a piece of iron away from a magnet, we encounter a resistance; here too we know that resistance is a force, — magnetic force due to magnetic energy. In both these instances the resulting force exists independently of the motion against it; but it often occurs, as in all frictional resistance, that the force is called into play by the motion which it opposes. Resistances which are plainly due to well-recognized forces, such as weight, electrical and magnetic forces, and so on, we do not need to consider

further; but it may be well to inquire into the nature of the resistance of friction. We shall find that this appears to be due to an opposing force arising from the elasticity of the materials involved, and ultimately results in the acceleration of particles of bodies, producing heat energy.

External Friction of Solids. — The friction of rubbing solid surfaces is due to their roughness. When solid surfaces, even if smooth, are examined under high magnifying power, they will be seen to be covered with a multitude of scratches and projecting points. In rubbing surfaces these irregularities interlock. As the one surface is drawn over the other, the projecting parts bend each other backward, bringing into play the force of elasticity. As the motion progresses, these parts bend sufficiently to slip past one another, or the bodies are more or less forced apart. Thus the resistance of solid friction is due in part to the force produced by elastic energy locally called into action at the points of contact. The energy expended in maintaining motion against friction is thus converted directly into energy of elasticity; then as these interlocking points slip past one another, they enter into vibration, the energy of which is soon transformed in turn into heat. This transformation takes place, in large part, within the substance itself through its imperfect elasticity, that is, through the presence of internal friction, the nature of which will be presently described. Another portion of the vibration energy is transmitted (sound waves) to surrounding bodies and is there similarly transformed into heat.

In the rubbing of two solid surfaces, the interlocking portions are more or less broken and scratched. These processes consist essentially of the separation of portions of the solid against the action of cohesion energy, as will be presently shown, and the energy thus expended remains in part as cohesion energy, and is partly reduced through vibration and imperfect elasticity to heat.

Two rubbing surfaces are ordinarily held in contact by weight or some external force; but when the surfaces approximate to perfect planes, or at any points where the contact between the two is very close, the force of cohesion undoubtedly comes into play by holding the surfaces still more firmly together, thus increasing the friction. At such points of intimate contact, internal friction undoubtedly plays a part.

Internal Friction of Solids. — A *hypothetical* explanation of this phenomena is as follows: The molecules of a solid are in rapid but restricted vibration irregularly in all directions. The kinetic energy of this vibratory motion constitutes the heat present in the body. Imagine a geometrical plane to be passed through the solid body. Particles from either side of the plane are continually passing through to the other side, coming into collision with particles of that side, and returning usually to their original place. Suppose now one part of the solid is being drawn over the other on this intersecting plane. For convenience suppose the plane to be horizontal. Then particles from the upper half will be continually passing into the lower half, carrying with them, on the average, the onward velocity of the upper half. On the average, these particles through their contact with those in the lower half will give up the energy corresponding to this onward velocity, and will return to the upper portion without it, and while there again take it up. There is thus a continuous passage of energy from the moving portion to the under portion which we will consider as stationary, that is, as the point of reference for the motion. This transferred energy will, in the lower half, assume the form of heat, since it will result in the increase of the kinetic energy of the particles. Although the velocity from which this energy arises is wholly in one direction, it will soon become irregularly distributed throughout the mass, as this is fixed in position. The energy thus becomes irregularly distributed

kinetic energy of the particles, and thus is of the form of heat.

Equally the particles below the plane are passing through to the upper side, and returning with an increase of energy acquired in virtue of the onward motion of the upper portion, and at the expense of the energy maintaining the motion; this added energy is at once converted into heat through collisions, and thus more of the acting energy is transformed.

When the great number of the interchanging particles per unit area, which the kinetic theory of heat requires, is considered, it is easy to understand how a very large expenditure is necessary to maintain such motion in a solid. In brittle solids, this motion can be carried to only a very slight extent, owing to the occurrence of fracture. In imperfect and ductile solids, and in general in all solids as they approach their melting-point, internal motion can be carried further without rupture, and the internal frictional resistance diminishes.

Since the energy maintaining the motion is expended in continuously accelerating the motion of the particles in the direction of motion, it must be exerting a force in that direction; also since the acceleration is imparted to the particles through their elasticity, there is a force of elasticity tending to retard the upper and to accelerate the lower (fixed) portion. This action constitutes a part of the internal friction of solids. Another part of the internal friction of solids — or more properly another way in which energy is expended in maintaining the motion referred to — arises through cohesion. Every particle of a solid is under the action of cohesion energy, which tends more or less strongly to make it approach each of its neighbors. On the average, the total force is the same in every direction upon any one particle, so that the resulting force is zero; but when motion of one portion of a solid takes place over an adjacent

portion, as in the above example, each particle above the separating plane is torn from its association with the particles behind it below that plane, and is brought into equally intimate association with those particles to which it is approaching and which also lie below the plane. So long as the average distance of the particle from its neighbor remains constant, the mere change of position requires no resulting expenditure of external energy; since the energy necessary to separate the particles against cohesion is exactly equal to that restored by cohesion as the particles approach others; but the recession from one particle and the approach to another, under a force which varies inversely as some function of the mutual distance, will in general set up in the particles, irregularly directed vibrations whose energy will add to the other heat energy of the body. Thus further demand will be made upon the energy-source which is maintaining the relative motion of the two parts of the body. This constitutes the remainder of the internal friction of solids.

If the solid is torn apart at such an imaginary plane; that is, if the two portions are separated so far that the force of cohesion is no longer sensible, then a part of the energy so expended is stored up as cohesion energy, to be restored if the parts are at any time brought near enough together for it to act. Another part is transformed into energy of vibration and thence into heat as just described.

Internal Friction of Fluids. — In liquids and gases the nature of the internal friction is substantially that of solids, except that the cohesion is nearly or quite insensible in its influence. There is also the difference that the particles do not oscillate about a mean fixed position, but wander with more or less rapidity and freedom throughout the volume of the substance. This, however, makes no important difference in the nature of the phenomenon.

Gases and liquids oppose a still further resistance to the

motion of a solid body through them, by reason of their displacement by the moving body. To set the displaced mass of fluid in motion requires an expenditure of energy. The kinetic energy thus imparted is rapidly reduced from a more or less irregular molar form to the irregular molecular form of heat, through the internal friction of the fluid. Also there is the hydrostatic pressure to be considered. When the submerged body is at rest relatively to the fluid, the pressure is equal in all directions at equal depths, if due to weight, but if the body is in motion, the pressure upon it in the direction of motion is lessened in proportion as the velocity is greater. For the energy causing the pressure acts only at a definite space-rate and can, therefore, accelerate the fluid as it follows up the moving solid at only a definite maximum rate. Part of the primary energy being thus expended in accelerating the motion of the fluid, less remains to exert force upon the moving solid; and when the acceleration of the fluid absorbs the primary energy at its full space-rate, then the hydrostatic pressure in the direction of the motion of the body ceases entirely. Thus the resisting hydrostatic pressure upon a solid moving in the fluid is zero when the motion is nil, and increases to a maximum when the motion is sufficiently rapid to bring about the condition above mentioned. For higher velocities the pressure remains constant.

There is also, in the motion of a solid through fluids, the surface friction, which is of the same general character as the surface resistance between solids.

Stress, Attraction, Repulsion. — When we consider the action of energy in producing tendency to acceleration with reference merely to the body acted upon, we ordinarily employ the term *force* to represent the action; when, however, the energy is acting to cause two bodies to approach or recede from one another and we are considering the action upon both of two bodies, we call it a stress, that is, we say

there is a stress between the two bodies; or if the energy is acting upon two parts of the same body, we speak of a stress in the body. If the action tends to move the body or particle away from the supposed location of the energy, we call the force a pressure; if the action is regarded as tending to move the body or particle towards the supposed location of the energy, we call the force a tension. When we regard the energy as tending to move bodies or particles further apart, the stress is called repulsion or sometimes pressure. When the stress tends to make two bodies or particles approach one another, it is called attraction or sometimes tension. A pressure may be called a push and a tension a pull. We speak also of a force of attraction when we are considering only one portion of the attractive stress, that is, the action upon one of the bodies or particles concerned; and similarly we speak of a force of repulsion. When elasticity is in operation, if there is compression, the stress is composed of equal pressures in opposite directions upon the two bodies or particles involved; or, if there is extension, the stress consists of equal attractions. It may be added that most forces appear to be of the nature of elastic force or stress, and it is generally conceded that force can be conceived of only as a pressure.

When we consider the action of gravitation upon bodies, we find no direct evidence of any stress in which the bodies take part; that is, when the gravitation energy is accelerating a body, it is exerting force upon that body, but there is no evidence that the body is exerting a force against gravitation; in fact we distinctly conceive it not to be doing so. Yet no hypothesis of gravitation has been framed which accounts for the gravitation of bodies without ascribing to the particles of the gravitating bodies, and also to the substance causing the gravitation, the property of elasticity. This fact points not only to the possibility suggested by other considerations, that matter does not inherently possess

the property of gravity, but that this property attaches only to portions of matter which have that energy associated with them which is necessary to constitute them *bodies* (molecules) as distinct from mere matter (cf. Chaps. VII. and XI.).

The use of the phrases, "a body attracts another," or "a body repels another" and similar ones, is somewhat misleading, since the phrases imply that the cause of the attraction or repulsion is resident upon the bodies, whereas in general we *do not know* the locality of the energy. They cannot, however, be excluded from use, and it therefore becomes necessary to remember that the foregoing implication *may* be contrary to fact.

Force as an Index of Energy. — Force being the action of energy, its presence is of course sufficient evidence, at any time, of the existence of a form of energy causing it. We need, therefore, no further proof of the presence of energy than the occurrence of some phenomenon which we know to accompany the presence of force. It is not, however, to be forgotten that our fundamental proof of the existence of force goes back to the test as to whether acceleration would occur under the given condition in a free body.

Remarks upon Force. — It is of the utmost importance for clear thinking to fix in the mind the fact which the proposed definition of force (p. 41) brings out; namely, that force is only a manifestation of energy. It is essentially secondary in its nature. Force can have no existence independently of energy. It cannot produce energy or destroy it. It is not a thing which can be transformed or which has any existence in the sense in which matter and energy exist, as indicated through the principles of the conservation of matter and energy. For brevity, it has been customary to speak of forces and bodies as doing certain things or performing certain actions. But it must be remembered that the thing which does or performs is really

the energy and not the force or body. It is the energy exerting the actions called force and work.

The utmost confusion prevails, in physical works of all kinds and still more in ordinary language, in the use of the terms *force* and *energy*. Unhappily our text-books of physics do little to remedy this state of affairs, and more often add to the confusion than relieve it. This is a natural inheritance from the state of physical knowledge of a half century or more ago. At that period various "Physical Forces," so called, were clearly recognized. These were, Motion, Heat, Electricity, Light, Magnetism, Chemical Affinity, and "Other Modes of Force" [Grove]. The idea of energy had not then been adopted, and what we now call the energy of a moving body, that is, its kinetic energy, was called "the force of motion" of the body. When such a moving body struck a resisting object, "the force of the blow" was the expression used to refer rather to the *energy* of the striking body than to the *force* proper which it exerted while it was being checked in its motion by the resistance of the object. Thus even after the distinction between force and energy began to be recognized, and the principle of convertibility to be apprehended (*e.g.* the transformation of heat into force of motion, kinetic energy, or *vice versa*), there still remained a great degree of confusion. This arose both from a lack of acquaintance with energy and force, and from the want of a clear definition of either. The confusion at the present time results almost wholly from the latter cause; for with a correct definition of force, it entirely vanishes.

The comprehension of physical phenomena becomes relatively simple and satisfactory if we accustom ourselves to thinking: first, of matter as absolutely inert, and therefore as incapable, of itself, of exerting force or any action whatever; second, of energy as a power conferred upon matter either by the bestowal of motion to it (cf. Part II.) or in

some unknown way, and as that to which all changes in the appearance or properties of inanimate things are due; and third, of force as an action of energy.

Since force is an action of energy and not something which in itself possesses a power to change the state of motion of bodies, confusion in the minds of beginners might be avoided if the term *force* were to be employed in the passive rather than in the active sense. Thus, we should speak of force as being exerted upon or against an object rather than as acting upon it; of motion, as arising from the existence, or presence, or application of the force rather than from the action of a force. This minor point is well worth the attention of those who are teaching the elements of the subject, and a little thought will show that no more circumlocution is necessary in employing the passive than the active form of expression, although it is by no means easy for one who has become accustomed to the latter to change to the former. To one who has at all mastered the subject, the phraseology is comparatively, though not entirely, unimportant. The difficulties of acquiring a clear conception of physical ideas are not so small that teachers can afford to overlook this point, even though at first sight so insignificant; indeed an imperfect understanding of the term *force* and of its relation to energy is perhaps the greatest stumbling-block in elementary physics.

CHAPTER VI.

KINETIC ENERGY, KINERGETY.

Apparatus. — The law of kinetic energy may be illustrated, as has been stated, by the collision of elastic balls. The method however is not general, since it deals only with bodies of nearly perfect elasticity. Also it is hardly sufficiently simple for elementary demonstration. Many other methods might be suggested, but the following has some advantages.

The apparatus consists chiefly of a spring buffer so designed that it can take up substantially all of the kinetic energy of an object impinging perpendicularly upon its surface. This energy is stored up as energy of elasticity in the spring of the buffer, and can be subsequently imparted in the form of kinetic energy to any free body resting against the buffer by merely releasing the spring. One form of the buffer may be briefly described thus. Two upright metal plates ten or twelve inches apart and one or two inches square are fastened to a base. Through guiding holes in these plates passes a straight steel rod or hard-brass tube about eighteen inches long and as light as is consistent with stiffness. The forward end of this rod carries a round, flat, circular plate an inch and a half in diameter, preferably of steel. This head-plate projects about four inches from the forward guide. Fastened to the rod just behind the forward guide is a collar, and extending from the collar to the rear guide and closely surrounding the rod is a spiral spring of well-tempered steel. Behind the rear guide is some sort of a clutch which shall catch and

hold the rod as it passes through the rear guide by any push upon the head-plate. The spring tends to force the head-plate and rod forward until the collar strikes the front guide, and to hold it there. The clutch must act so promptly and effectively as to hold the rod at sensibly the furthest point which it reaches, and must be so arranged that it can be completely released when desired, giving the rod entire freedom to move forward until stopped by the collar. One form of clutch which is sufficiently effective consists of a taper wedge which shall slide point downward, back of the rear guide, by its own weight, the slope of the wedge bearing always against a pin projecting at right angles to the rod. A ring-clutch such as is used on many arc lamps is better, but rather more difficult to construct. The spring must be of such stiffness as to be compressed three or four inches by the most powerful blow to which the buffer is to be exposed.

The objects whose kinetic energy is to be experimented with are best made in the form of balls, and are suspended by a cord of any desired length, or better by two cords diverging upward. The point of attachment of these cords at their upper ends should be such as to allow the body to rest lightly against the head-plate of the buffer when the clutch is released and the body hanging free. If such a suspended body be drawn backward in a vertical plane through the rod of the buffer and allowed to swing free, it will strike the face of the buffer and be brought to rest by the spring. Its kinetic energy relative to the buffer will have been entirely given up. This energy will have been mainly transformed into energy of elasticity in the spring, and this will be held stored up there through the action of the clutch which prevents the spring from giving it up. Small portions of the energy will have been otherwise expended. Some will have been transformed into heat through friction of the moving parts; some into sound

vibration; some into cohesion energy and heat in the striking body as well as in the buffer, owing to imperfect elasticity; and finally, some into energy of elasticity in the body itself. The last-named portion will by its action against the face of the buffer push the body away, causing a slight rebound. The energy of this rebound must be deducted, in careful quantitative experiments, but will ordinarily be so small as to be negligible in class-room work. The same remark holds with reference to the other portions of energy which are lost. Of course if the final position of rest of the buffer after the blow is not such that the object when in contact with it is directly below its point of suspension, the object will swing back as a pendulum. In more careful work, therefore, it is necessary to adjust suitably the position of the buffer to avoid this slight disturbance.

After such a blow any other object may be substituted for the first, and the clutch then released. The elastic energy of the spring will then impart kinetic energy to the object, slight losses occurring in the transfer similar to those above named. To show approximately what the loss of energy is in the process, a given object moving at a definite velocity may be allowed to strike the buffer, and the energy thus transferred to the buffer may be afterwards restored to the same object. The object should leave with the same velocity as that with which it struck. The difference in its velocity, or rather the corresponding difference in the amount of its energy, the computation of which will be shown later, gives the loss in the operation. The amount of this loss, however, varies with the amount of energy employed, and the nature of the substance of the striking object.

The relative velocities on striking and upon leaving the buffer can be determined by the ordinary experimental law of the pendulum. The square of the velocity at the lowest

point of its swing is proportional to the vertical distance through which the object descends. Also, if the suspension is several times the length of the arc through which the object swings before or after striking, the vertical distance through which it descends is nearly proportional to the horizontal distance through which it swings.

The foregoing apparatus, even without the application of corrections for minor losses, will serve sufficiently well for the following illustrations. It will of course give a much closer approximation with the use of the corrections. It is sufficiently exact to justify the assumption made in the following demonstrations that (sensibly) all the kinetic energy of one body may be expended in producing kinetic energy in another body. The transference is, to be sure, not a direct process, but this in no way concerns the correctness of the proposition; moreover, in fact, as may be seen by an examination of all imaginable cases of the transference of kinetic energy, direct transference never occurs.

Quantity of Kinetic Energy. — In defining quantity of kinetic energy we must proceed directly from the fundamental definition. Since that definition bases the recognition of energy upon change in the state of motion of bodies, quantity of energy must be primarily defined through amount of change in state of motion. As all change in state of motion may be resolved, as shown in the chapter on Motion, into acceleration from a state of rest, such acceleration is naturally adopted as the basis of primary measurement. Also as free motion is the simplest case of motion, the fundamental measurement must be through the acceleration of free motion of the body. An axiom which must lie at the basis of the measurement is then: —

To accelerate equally from rest the free motion of a constant body at different times, equal quantities of energy must be imparted to the body.

This axiom affords a test for, or defines, equal quantities of kinetic energy.

By a *constant* body or *the same* body will be meant a body which is undergoing no discoverable change in any of its properties during the time of experiment. Pieces of metal, quartz, glass, etc., are common examples of sensibly constant bodies. By a *standard* body is meant some constant body selected and preserved for reference. Such standard bodies are the International Kilogramme and the Imperial Pound, represented in practice by their metallic copies.

The first proposition to be proved in studying the law of kinetic energy is this: —

The same body moving with equal velocity at different times possesses the same quantity of kinetic energy.

This may be illustrated by the apparatus described in a preceding paragraph. Let a constant body A give up all its kinetic energy to the spring buffer; and let this energy be subsequently given out in accelerating from rest a second free body B. In successive trials, the resulting velocity of B will be found the same whenever the velocity of A on impact is the same. This observation is confirmed by general experience.

Capacity for Kinetic Energy: Kinergety.[1] — The second proposition to be established is: —

Any given body possesses a definite capacity for kinetic

[1] The importance of enforcing the concept of capacity or receptivity for kinetic energy, and the awkwardness of the phrase itself, justify the introduction of a new term. The word *kinergety* seems well adapted to the purpose. It will be used both in the abstract sense, to denote the idea of *capacity* for kinetic energy, and in the concrete sense, to denote the *quantity* of this capacity; following in this respect the analogy of energy and many other terms. The etymology of the word is sufficiently obvious.

Certain other terms similarly derived, and perhaps also new, may be suggested as being of possible service. These are: —

Energia = energy, or, perhaps better, quantity of energy. Such a term

energy, that is, a definite kinergety; and this is constant in amount so long as the body is constant.

The term *kinergety* will be employed to denote *capacity for kinetic energy* (cf. Chap. XI.). By the possession of a definite kinergety is meant, that a body moving at different times with the same velocity will possess the same amount, neither more nor less, of kinetic energy. In the case of many constant bodies this proposition is capable of direct proof or illustration by the apparatus described in the former paragraph. In all cases it may be inferred from observed phenomena.

Different bodies in general possess different kinergeties.

This third proposition is obvious in cases where we build up bodies from smaller parts. It may be in general proved in the same way as the preceding proposition.

This fact that each individual object or body (that is, every portion of every substance) possesses an individual and fixed kinergety, is one of most fundamental importance. The kinergety *is or appears to be* a permanent property of substance, and appears to inhere in each individual portion quite independently of the existence of any other portion. There is no evidence that its presence is due to anything external to the substance or to anything associated with it. On the other hand, so far as we can discern, to annihilate this capacity would be to annihilate the substance simultaneously. But however natural this inference

would be very serviceable in physics, and its adoption would be in line with recent convention with regard to such terms as induction and inductance, etc.

Energic = possessing energy, — an adjective form now obsolete but well adapted for use in physics where energetic would be unsuitable because of its significance of *capacity or ability to exert* energy.

Kinergy and *kinergia*, to denote respectively kinetic energy and quantity of kinetic energy.

Kinergic = possessing kinetic energy.

Kinerg = a unit quantity of kinetic energy; *e.g.* one-half of the kinergia of one gramme moving with a velocity of one centimeter per second.

may seem, we must be very slow to assert its truth. That the capacity is not something superposed upon the substance and capable of being removed from it, cannot be held to be yet demonstrable.

Kinergety *vs.* **Mass.**—The kinergety of substance is clearly a property distinct from all others. Whether it is also a property of matter, or whether it is the consequence of a different property in matter, such for instance as the occupancy of space, need not now be considered (cf. Chap. XI. and Part II.). In either case, neither kinergety nor any possible cause of it, forms properly any part of the meaning (either denotation or connotation) of the term *mass.* By almost universal consent, mass denotes *quantity of matter,*—and nothing further. For certain good reasons presented in a later chapter, we assume that the quantity of matter—mass—in a body is directly proportional to the body's kinergety. But this in no way renders the concept of kinergety a part of the significance of mass. As much should we be justified in assuming that weight and kinergety have anything in common in their meanings because we find them to be proportional to one another in quantity; or that weight and mass are in any way identical because proportional. This absolute unrelatedness of the concepts expressed by the terms *mass* and *kinergety,* respectively, is very generally ignored in the presentation of physics, as is likewise the fact that the proportionality between mass and kinergety (quantity of matter and of capacity for kinetic energy) is an assumption. There is perhaps no point in the elements of physics upon which more confusion and haziness of ideas prevails.

In the following pages it must be remembered wherever mass is used, that this is measured only through the *assumption* of its proportionality to kinergety or to weight. The methods for these measurements will be duly presented.

Equal Kinergety. — *Bodies have equal kinergety when, with equal velocity, they possess equal quantities of kinetic energy.*

This follows directly from the definition of the term. Evidently, then, we may make a second body which shall have a kinergety equal to that of a given constant body which we may call a standard body, by taking from or adding to its substance until, by experimental trial with such an apparatus as that already described, it fulfils the requirement of the foregoing paragraph.

Graded Set of Kinergeties or Masses. — Having thus constructed two or more bodies, A, of equal kinergety, we may proceed to make a set of bodies whose kinergeties shall be to one another in any graded series that we may desire. We have, first, to make a single body, B, which when moving with a given velocity, shall be capable of producing the same acceleration upon some constant body as is produced conjointly by two of these bodies A of equal kinergety when moving with the same velocity as B. This new body, B, will have a kinergety twice that of each of the bodies A. Continuing in this way we may produce any multiple kinergety, or any desired submultiple. Such a set of bodies of graded kinergeties we have in our ordinary "set of weights"; although the latter are constructed by a different process which will be described later. Assuming that mass is directly proportional to kinergety, this set of bodies will also constitute a correspondingly *graded set of masses* and is often so called.

Measurement of Kinergety. — From the definition of kinergety it follows that relative kinergeties are simply proportional to the relative amounts of kinetic energy contained in the respective bodies when moving with equal velocities. A primary method[1] of measuring relative kinergety of different

[1] A primary method is one which is based directly on the definition of the quantity to be measured and is in no way dependent on any other mode of measuring the same kind of quantity.

bodies may therefore be laid out along the experimental lines already described. By the spring buffer or other suitable device, let the body X of which the kinergety is to be ascertained expend all its kinetic energy in accelerating from rest the free motion of a constant body B. Let V denote the resulting velocity of B. Find by trial what combination of the bodies of the graded set above described will impart to B this same velocity V, the velocity of the combination being the same as that of X. The kinergety of the combination is the sum of the kinergeties of its parts, and as these are known, the whole is known, and this is equal to the kinergety of X.

Although no such method would yield accurate results in practice, because of experimental difficulties, yet no method giving results inconsistent therewith could be accepted. Hence, although "weighing" is the customary practical mode of measuring kinergety (and mass), it is adopted only after rigorous proof that the relative weights of bodies at a given point of the earth's surface are sensibly proportional to their kinergeties, a proof to be presented later.

A little consideration will show that this primary method for kinergety measurement will also serve for primary measurement of kinetic energy.

To secure uniformity in measurements of kinergety at different times and places it is obviously necessary to adopt a standard of kinergety. Clearly, the only way to do this is to select some suitable, unchangeable body, and preserve it with the utmost care. Such a standard is afforded by the "International Kilogramme" and by the "Imperial Pound." Quantities of kinergety would be numerically expressed in terms of this standard quantity as a unit,[1] or in terms of some designated multiple or submultiple of it, ac-

[1] The distinction here indicated between a standard and a unit applies to measurement of quantities of all kinds, and should be carefully observed. Standards of many of the so-called "derived" quantities, such as kinetic

cording to convenience, *e.g.* in kilogrammes, grammes, centigrammes, etc., or in tons, pounds, grains, etc.

Law of Kinetic Energy. — The first proposition under this law is: —

The kinetic energy of different bodies when moving with equal velocities is proportional directly to their respective kinergeties.

This follows directly from the definition; since the kinergety of a body is defined as being proportional to the quantity of kinetic energy which the body will possess when moving with a specified velocity. The second proposition is: —

The kinetic energy of the same body moving at different times with different velocities is proportional directly to the square of the velocities.

This proposition requires experimental demonstration. It may be illustrated by the spring-buffer apparatus. Let a given constant body A expend all its kinetic energy in accelerating from rest a second constant free body B. Let this operation be repeated with velocities of A at impact successively in the proportion 1 : 2 : 3, etc. Note the corresponding velocities of B. Then find by trial with the graded set of bodies above described the kinergeties successively necessary to produce again upon B the various noted accelerations. The velocity of the trial bodies being the same at impact in all trials, the kinergeties requisite will be found to be in the proportion of 1 : 4 : 9, etc. But from the first proposition in this section, the kinetic energy of these trial bodies, their velocities having been the same, will have been in the ratio of their kinergeties; namely, 1 : 4 : 9, etc. These quantities of energy must have been equal to the respective quantities of energy exerted by A

energy, heat, electric quantity, magnetic field, etc., are not represented by any material standard, while others are so represented, such as electrical resistance and electromotive force.

when its velocity was successively in the ratio 1 : 2 : 3, etc. Hence the kinetic energy of A must have been as 1 : 4 : 9; that is, proportional to the square of its velocity.

Combining these two propositions it follows that the kinetic energy of a body is proportional directly to its kinergety and to the square of its velocity.

The velocity V is the instantaneous velocity along the path of the body, and the corresponding quantity of energy E is the body's kinetic energy relatively to any fixed point on the path, or to any point fixed with reference to the path. The reference point for both path and point must be the same, but may be any chosen point whatever not located upon the body itself.

If K be used to denote the numerical value of the kinergety of a given body (measured as above described), and V to denote its velocity, then by this law the kinetic energy of the body would be proportional to the product KV^2. If we were to assume as the unit quantity of energy the quantity possessed by a body of unit kinergety when moving with unit velocity, then KV^2 would give the numerical value of the kinetic energy of the body in the above case. But for reasons to be stated later, the unit of energy which has been universally adopted in practice has twice this magnitude, so that the factor $\frac{1}{2}$ must be introduced, and the expression becomes $\frac{1}{2}\,KV^2$.

Assuming, as we do, that mass and kinergety are directly proportional to one another, then if M denote the mass of the above body, the kinetic energy of the body will also be given numerically by the expression $\frac{1}{2}\,MV^2$.

Primary Measurement of Kinetic Energy. — The method for this purpose would consist in measuring the kinergety of the given body. The body's kinetic energy when moving with any known or measured velocity would then be equal to one-half the product of the kinergety into the square of the velocity. An alternative method would con-

sist in ascertaining by trial, what combination of the set of bodies of known kinergeties would, when moving with the velocity of the unknown body, have the same amount of kinetic energy.

The customary scientific unit of kinetic energy is one-half of that possessed by one gramme when moving with a velocity of one centimeter per second. It is called the *erg*, but might better be named the *kinerg*, as suggested in the footnote on page 60.

Attention should be here called to the proposition that kinergety is proportional to *weightal*, a quantity to be later defined.

CHAPTER VII.

ENERGY (Continued).

Transformation of Energy. — It may be shown experimentally by means which are described in text-books of physics, that, —

Energy may be changed from any one of its forms into any other; that is, may be transformed, either directly or indirectly.

This is known as the principle of the transformation of energy. Illustrations have been given on pages 25 to 37, and others will be found in the following pages.

Conservation of Energy. — The foregoing qualitative principle is supplemented by a quantitative one known as the Conservation of Energy, which may be thus stated: —

When any quantity of energy of one form disappears, a precisely equal quantity of energy simultaneously appears in some other form or forms.

These two principles are sometimes united and spoken of as the "Principle of the Transformation and Conservation of Energy." They underlie most of the recent advances in physical science. Their great scope will be obvious when we reflect that every phenomenon is due to a transference or transformation of energy.

The principle of transformation, as it was historically arrived at, was an inference from experimental evidence. Sundry diverse conditions of bodies were observed under which they had certain powers in common, such as the power to change the state of motion of free bodies, and to move bodies against resistance. These vaguely understood powers were in reality diverse in their nature. Some of them were what we now call forces; some were what we

now call energy; some, even, were substances. They ultimately came to be designated "The Physical Forces." Each of these "forces" had its own name (p. 53), and at first no common relation was discerned between them. But it was gradually perceived that a relation existed which is now known as transformability. Thus it was discovered that, under certain conditions, motion of bodies could be changed into heat or *vice versa;* that electrical currents could be made to produce either motion of bodies or heat; that gravitation could produce and maintain the motion of a body; and that, if this moving body were kept in motion by gravitation against friction, heat would be developed at the expense of gravitation. As the knowledge of such facts developed, it gave rise to the statement of a principle known as the "Correlation of the Physical Forces." Further study showed that when one of these so-called forces produced another, there was a quantitative as well as a qualitative relation; so that when a transformation occurred, a definite quantity of one kind disappeared, and a definite quantity of another was simultaneously produced. These quantities were not always the quantities or powers which had been vaguely termed the forces, but were intimate functions of them; and the quantities which were actually transformed came eventually to be called by the general term *energy.*

It is manifest that the establishment of the quantitative law of conservation of energy demands the ability to measure quantities of energy of different forms. If in all its diverse forms, energy were directly recognizable and measurable as a single quantity, the enunciation of the law of conservation would doubtless have been made fully a century earlier. For the general metaphysical idea of the equivalents of cause and effect, and of the impossibility of creating something out of nothing, as well as the denial of the possibility of a *perpetuum mobile,* found acceptance with philosophers at an even more remote period. In fact, how-

ever, not one of the forms of energy is quantitatively measurable directly; each must be obtained as the product of two or more factors of different kinds, which must be independently measured. What the factors were whose products should constitute, for example, gravitation energy, electrical energy, heat, etc. (as we now call them), was far too obscure a matter to be clearly apprehended even by the most penetrating minds, in the state of experimental science preceding the first quarter of this century. Through the progress of experimental science it gradually became manifest what the factors were whose product measured that quantity which was constant throughout all its transformations in the various phenomena.

Thus, it was early recognized that the effectiveness of a moving body, or its power, or what we now call its kinetic energy, was proportional jointly to the mass and the square of the velocity.

Again, it was found in the case of a body of weight W and mass M, falling freely through a vertical distance H, starting from rest, that its kinetic energy, $\frac{1}{2}MV^2$, was always equal (employing proper units) to the product WH; V being the velocity at the end of the fall. The quantity WH may be called the quantity of gravitation energy exerted, just as $\frac{1}{2}MV^2$ is the resulting kinetic energy. It was found that when the power of a descending body of weight W was expended against friction, either by stirring water or by rubbing together metal surfaces immersed in water, that the quantity WH was equivalent always to a definite amount of heat, as represented by the product of the weight of the water heated, multiplied into its rise of temperature.

It was ascertained that in an electrically heated wire whose heat was imparted to a quantity of water, the product CVT or C^2RT was always the equivalent of, or rather was capable of producing, a definite quantity of heat, meas-

ured as just stated; where C is the current, V the difference of potential (electrical pressure) between the ends of the wire, R the electrical resistance of the wire, and T is the duration of the current.

It was discovered that a definite quantity of heat, measured as above indicated, was invariably produced by the combination of definite masses of chemical substances; for instance, that a gramme of carbon burning in oxygen to form carbon dioxide (carbonic acid) produced a definite number of units of heat. It was found that when a gramme of zinc was completely oxidized, the heat evolved, allowing for incidental operations, was the same whether the action occurred directly or through solution in sulphuric acid. But if the latter operation took place in an electrical battery, then the quantity of heat generated was less in direct proportion to the amount of energy otherwise expended by the electrical current.

In these illustrations, gravitation, electricity, heat, and chemical action, would originally have been called "Physical Forces," but according to the modern terminology, gravitation is regarded rather as the name of a property exhibited by bodies and due to a form of energy, than as the name of the force corresponding to this property. The latter is ordinarily termed either the force of gravitation (between any two bodies), or gravity (between bodies and the earth). That which is transformed in the above experiment is neither gravity nor force, but the *energy* of gravitation. Heat is not a force, in the modern view, but a form of energy. Chemical action is now ascribed to a form of energy. Electricity is at present regarded as neither a force nor a form of energy. Whether it shall be regarded as a substance or as a (so-called) fluid, or as identical with the luminiferous ether, or whether it should be merely left as one of the things at present wholly unknown, is largely a matter of personal opinion among physicists. The quantity

CVT which is transformed in the above illustration is the *energy* of the electrical current.

From this statement it is easy to see how confusion would arise in physical literature as soon as force began to be restricted to its legitimate meaning, but this natural confusion was rendered far greater by the failure to perceive, or at least to state distinctly, that force itself is not a kind of energy. Many of the early specific definitions, which moreover are still current, gave it merely the vague character of "that which" can produce motion. This, unhappily, is a definition *not of force but of energy.*

The principle of the conservation of energy is a generalization from experience, and possesses a weight which increases as experience continues to confirm the proposition and deductions from it. Our greatest confidence in the principle is established not by the limited number of direct quantitative confirmations, so much as by the concordance between the multifarious applications of the principle and the corresponding observed facts.

The contradiction of the principle of conservation involves us at once in the assumption that energy can be created out of nothing, since the definition as well as the principle of the conservation of matter (cf. Chap. IX.) shows that energy cannot be a substance, and anything out of which energy can be created would, of course, under the principle of transformation, be classified as energy. But the assumption that energy could be produced from nothing would lead to immediate and endless contradiction between facts and the logical deductions from this proposition. It leads directly to the principle of "perpetual motion," which all experimental evidence flatly contradicts. The principle of conservation may then be regarded either as a proposition established by direct experimental evidence, or as one deduced from the denial of the possibility of perpetual motion — a denial in turn based on experience; or we may regard

it as established jointly by both of these bodies of experimental evidence. The first two views have been advocated with undue bitterness in the advancement of claims for personal priority. Laying aside this sort of dispute as in no wise pertaining to science, the third view is doubtless the more rational.

Criterion for Energy. — From a modern standpoint, the principle of transformation has become largely a matter of definition; for whatever shape the definition of energy actually takes, if it be a true one it must be consistent with the principle of transformation, and the real criterion to which everything must be submitted before it is allowed to enter the category of energy, is the application of this principle of transformation which may be stated as follows: —

Anything is energy which is capable of being transformed into any known form of energy, or into which any known form of energy can be transformed.

This criterion can be applied in the following manner, and in fact is always applied when it is desired to test whether or not a quantity or thing is energy. Inspection will at once show that the definition of energy adopted in this volume (p. 20) corresponds to a particular application of this general criterion.

Starting with one recognized form of energy, as for instance kinetic energy, we may proceed to discover a second form by finding something which can be transformed into kinetic energy. We may then discover a third form by finding something which may be transformed into either of the first two, or into which either of them may be transformed. Proceeding in this way, the facility of recognition of any new form of energy continuously increases under this criterion. We cannot fail to recognize any form of energy, neither can we include anything which is not energy; for transformability is the one essential test of energy. It may be noted, however, that the principle of

conservation does not affirm that *all* the energy of any one kind in the universe is convertible into any other form or forms.

Historical Sketch. — A brief sketch of the doctrine of energy and of the conservation of energy may serve to indicate more forcibly the present dominating position of the latter in physical science. For dates and for the names of those physicists who have contributed to this subject — and the list embraces all the leading physicists of this century — reference must be made to such an account as is given in Planck's "Erhaltung der Energie."

Beginning with the time when it was supposed to be necessary to apply power continuously to free bodies in order to maintain their motion, the progress was slow to the time, two centuries ago, when this was distinctly contradicted and correct views of the law of motion were enunciated by Newton. But even before that epoch the denial of the possibility of perpetual motion, or more properly of a perpetual motor, had been more or less clearly enunciated; the denial being based partly on *a priori* arguments, partly upon want of success in experimental trials. Although Newton seems to have had some conception of the idea of conservation in the action of machines, yet the want of anything approaching to prominence in his treatment of this subject seems to show that it occupied at least no important position among his views. And he certainly extended it to no part of physics as known in his time, beyond a limited portion of the domain of mechanics. He uses no term equivalent to energy, nor any term, other than a phrase, for mechanical power. Newton's idea of force was very distinctly stated, as shown in part in the definition quoted in Chapter XII., and it corresponded closely with present usage. His measure of force was the time-rate of change of momentum. During the century following Newton's work small progress was made in the development of the concept of energy

except in the domain of mechanics where the law of conservation of *vis viva* for elastic collision was established, and disbelief in the possibility of the *perpetuum mobile* was more than once clearly stated. That *power was lost* (in the sense of annihilated), through friction and other resistances, was still the prevailing belief. But that either power or the motion of matter could be produced out of nothing was for the most part emphatically denied; still, the state of physical knowledge was such that there was little or no thought of connecting by any common link diverse phenomena such as gravitation, mechanical power, light, heat, electricity, and so on. The end of the eighteenth century, however, saw the substantial beginnings of this broader view, in such far-reaching investigation as that of Rumford and Davy, and the first quarter of the present century witnessed important advances; not only did the knowledge of physical phenomena and laws steadily increase, but the notion of convertibility, and of the interdependence of phenomena gained strength. Heat, however, was still by many regarded as a form of matter. The nature of light was not understood, and the knowledge of the laws of electricity and magnetism was very incomplete. During the second quarter of the century, the state of physical knowledge was advancing at a rapidly accelerating pace towards the great announcement of the principle of conservation which the middle of the century was to bring forth. The perception of the fundamental idea of conservation, as well as the first clear statement of the principle, was based upon the denial of the possibility of perpetual motion. This denial rested largely upon metaphysical speculation, although supported by an array of experimental evidence confined almost wholly to the domain of mechanics. The extension of the principle of conservation to other portions of physics from such a basis at this epoch, however brilliant as an effort of human reason, and however fully confirmed by subsequent study,

cannot be considered to have been a rigid physical demonstration. It was, therefore, not until adequate strictly physical testimony had been afforded through the exhaustive experimental work of Joule that the principle of the conservation of energy, although previously formulated by Mayer, could be regarded as a legitimate part of physical science. From the date of the beginning of Joule's work (about 1842), the concept of the nature of energy, and the principle of its transformation and conservation, developed and were confirmed at an ever-increasing rate. Within twenty years, these had progressed to a rank, not second in importance even to the corresponding concept and law of matter. During this short time all laws and phenomena had been subjected to the closest scrutiny as to agreement with the principle of conservation, and immense progress had resulted both in the theory of physical science and in the discovery of new phenomena. So absolute is the confidence now accorded to the principle, that it is universally adopted, along with the conservation of matter, as the fundamental starting-point in all physical inquiry.

That the establishment of the principle should have awaited, as it did, a wider knowledge of physical phenomena is easy to see when we consider the complexity involved in the measurement of some forms of energy, and the radical difference in the character of the quantities measured and the phenomena involved. No form of energy is directly measured, as for instance we measure length or weight, but each form is indirectly measured. That is, each form is measured as a product of two or more separately measured factors or quantities. Until these quantities and the means of their measurement were understood, it was impossible to measure the corresponding form of energy. This has already been stated in some detail at page 68. Thus gravitation energy is measured for objects near the earth's surface by the product of weight into vertical distance

moved through. The energy of electrical currents is measured by the product of the current, into the difference of potential between the two points considered, into the time of duration of the current. The relative energy of two charges of electricity is measured by the product of the force into the distance through which the charges are moved towards (or away from) each other, under that force. Heat is measured by the product of the mass of water heated, into the number of degrees of temperature through which this rises when the heat is applied to it. Radiant energy is measured by transforming it into heat; and so on for other forms. The briefest consideration will show that while it might be perceived that the power of a moving body was transformable into heat, or into energy of electric currents, and so on, still any quantitative relation might escape detection for want of knowledge either as to the nature of the quantities to be measured, or as to how they should be measured. And not only was this great obstacle to be overcome, but confusion and delay arose from unfortunate discussion which involved in a large measure merely questions of terminology. Thus a most bitter controversy was long conducted as to what should be held to be the true measure of *force*. Descartes and his followers contended that force should be measured by the rate of change of momentum. Leibnitz, strongly supported, held that force should be measured by what we now call the work done, or kinetic energy produced, by its action. The discussion was, however, not one of terminology merely, as might appear from this statement. It involved also the question as to which of these two was the fundamental quantity to be dealt with. In respect to this latter point Leibnitz was, as it now appears, undoubtedly right; and following him Helmholtz and most of the other prominent physicists dealing with this subject up to about 1860 used the term *force* (*Kraft*) in a sense entirely equivalent to that in which

energy is now employed. But ultimately, prolonged usage, supported chiefly by the classic writings of Newton, prevailed, and the term *force* was restricted to its present significance. The term *energy*, first proposed by Young in 1832 as a name for *vis viva* (MV^2), was adopted and is now universally employed to represent what Leibnitz indicated by force (*Kraft*) as well as the extension of that idea. That this diversity in the significance attached to the word force (*Kraft*) by most competent authorities should have led to very great confusion in the literature of physics, is most natural. And though there is at present entire unanimity as to the employment of the term, this confusion has not yet disappeared from even recent text-books of physics and still less from colloquial use.

The commanding position of the doctrine of conservation of energy in physical science can hardly be too strongly insisted upon. It not only furnishes the sure criterion for the discrimination of energy, but it affords the inevitable test to which any proposed operation, or any supposed discovery, invention, or law of the physical world must be subjected; and since all physical phenomena come under its scope, the deduction of these phenomena and their laws from this principle as a starting-point must be possible. Although for nearly forty years the principle has been so firmly established, each year shows almost revolutionary changes in the treatment of the laws of physics, arising from this point of view. Instead of regarding these laws as inductions from observed phenomena, thus passing from the particular to the general, they are more and more treated as deductions from the general principle of the conservation of energy. And fruitful as the principle at first was in correlating phenomena and promoting discovery, it has now proved hardly less effectual in unifying and generalizing the presentation of these laws.

Other Statements of the Principle. — The principle of con-

servation may be also stated in two other forms as follows. First, in any system of bodies entirely enclosed by a boundary, real or imaginary, through which no energy passes during the time under consideration, the total quantity of energy remains constant whatever phenomena take place within the system. Second, the total quantity of energy in the universe is a constant. This second statement is to be regarded as merely a special case of the first, since the universe, whether finite or not, comprises everything, and can therefore neither be receiving nor giving out energy. Both of these forms are open to criticism in ways from which the statement at page 67 is free.

EXAMPLES OF THE TRANSFORMATION OF ENERGY.

Let us now take a few specific instances of transformation and conservation of energy.

Gravitation and Kinetic Energy. — Kinetic energy can be transformed into gravitation energy by directing the motion of a body upwards from the earth's surface. Thus if a body be thrown upwards from the ground, it will continue to rise with retarded motion until its velocity becomes zero, that is, until its total kinetic energy has disappeared. If this is done under such conditions that no other changes take place except such minor disturbances or losses as may be allowed for, what becomes of the kinetic energy? If the body be now allowed to fall freely under its own weight, it will have acquired, when it has descended through exactly the same distance S through which it ascended, a velocity V, identical with that with which it started upwards. It will then have received exactly the amount of kinetic energy with which it started upwards. Whence has this energy come?

This experience is correspondingly true of the gravitation between any two portions of substance; the body and the earth are merely convenient examples, the more so because

of the great mass of the earth which renders its acceleration in such cases negligible. What inference are we to draw? In falling, the object acquires energy of a definite amount and at the same space-rate. In ascending, the object gives up energy equal in amount and (as may be shown from its uniformly retarded motion) at an exactly equal space-rate. We know also (p. 44) that during the ascent, as well as during the fall, and at all other times, there is a force exerted downwards upon the body; namely its weight. This continual and uniform action of weight is indicative of a *store of energy* always acting upon the body to produce this weight. The exact and invariable relation between the amount and rate of energy in ascending and descending, are inconsistent with any hypothesis which does not assume that the energy given out during the ascent is stored up and an equivalent imparted during the descent. It is natural to assume that the energy given up in ascent is merely imparted to this store, and that in descent an equivalent amount is returned from that store; for the body would give up no energy in rising *if not acted upon by the energy which causes its weight, that is, if it were not moving in opposition to a force;* since it is not accelerating any *known* portion of matter.

It is important to notice the evidence for this store or stock of energy. The weight of the body is precisely the same in its lowest as in its highest position, that is, before the kinetic energy has been converted into energy of gravitation. It is, therefore, not this *newly formed* gravitation energy which is producing the weight; and further since this added gravitation energy fails to give the slightest evidence of its existence by increase of the weight of the body, it must be an insignificant amount as compared to that store of gravitation energy which is producing weight. Again, a body at a point above the earth's surface will acquire energy in descending whether it has previously been in a

position nearer to the surface or not. To take an extreme illustration, a meteorite undoubtedly follows the same law of approach to the earth as any equivalent portion of terrestrial matter would if raised from the surface to the position of the meteorite. That is, the preliminary transformation of other energy into gravitation energy by raising a body from the earth's surface to a more elevated position is not, so far as we know, a preliminary requisite to the capacity of that body to acquire energy in falling. To put this in a more general way, we may say that so far as we have any knowledge of the state of the universe anterior to the present, this knowledge indicates that the average distance between the particles of matter forming the earth has been progressively lessening. This, apart from pointing to the long existence of the phenomenon of gravitation, shows that the energy which gives to bodies a tendency to approach the earth's surface preceded the present state of aggregation of the materials of the earth. Therefore it is not necessary to assume that in order for bodies at a distance from one another to have a tendency to approach, they should previously have been situated in a position of closer proximity.

The conception of the existence of a store of gravitation energy beyond the insignificant amounts arising from change of position of bodies is thus a reasonable inference. We may naturally inquire where this store of energy resides, and might seek it *first* in the body itself. But, we have not the least indication, through any change whatever, that the body is in any way modified by its removal upwards from the earth's surface. We should most naturally look for evidence of the existence of the stored-up energy, as the body rises, in increased motion of some kind in its particles; but motion of the particles of bodies produces several well-known phenomena, temperature, radiation, etc., and none of these are in the least modified. However closely we scrutinize

the body, there is no indication to be found either of the presence of the energy producing weight, or of the added energy which has been stored up through the increased elevation of the object.

Does the store of gravitation energy then exist outside of bodies, and is it perhaps distributed throughout space? This seems the only natural alternative hypothesis and is one which the mind very readily accepts. Still, beyond the facts of gravitation, there is not the least indication of the presence of such a universal store.

For a complete understanding of the treatment of the doctrine of energy, it is not necessary to choose between these hypotheses or to adopt any view whatever. The first of the hypotheses involves us in a further complication of accounting for the action of the energy resident on one body upon the energy resident on the other body at a distance from the first; since "action at a distance," without the intervention of some intermediate material agency, so vigorously disclaimed by Newton and by Faraday, continues to be regarded as an untenable supposition. We are obliged to infer some medium for the transmission of gravitation force. Under the hypothesis that the store of gravitation energy is uniformly distributed throughout space, no such supposition is necessary, so that the second hypothesis has perhaps the merit of greater simplicity. It is undoubtedly the one which more nearly represents the general trend of scientific speculation (cf. Part II.). It is, however, of the utmost importance to recognize the possibility of more than one hypothesis as to the locality of this energy. We must give ourselves the broadest latitude possible, and the greatest freedom from the bias which would result either from adopting an insufficient hypothesis, or from failing to perceive the possibility of an alternative.

If we bear in mind that this relation holds between any two bodies, at least within the range of our solar system, we

must infer that this store of energy, if such there be, must extend at least to the boundaries of this system. The existence of companion stars, apparently in orbital motion about each other, indicates the probability that gravitation also attends some other portions, at least, of the universe. We have, however, no direct evidence that gravitation extends throughout all portions of space, though it is perhaps difficult to imagine that its extent is limited.

To summarize briefly then we may say that we may infer the existence of a store of what we choose to call gravitation energy, perhaps located upon bodies, perhaps distributed throughout space, whose extent is at least as great as that of the solar system, and which acts continuously and uniformly on all portions of substance. This action produces between any two bodies a tendency to acceleration towards one another, such that if the portions are free and start from rest, they will be made to approach each other with accelerated motion, in a straight line joining their centers, and each will acquire energy relatively to its former condition of rest at a definite space-rate, which will be the same for both. That is, the energy exerts upon each of the two bodies an equal force, tending to make them approach each other along a straight line. This force is what we call the force of gravitation; or when the two bodies are respectively an object and the earth, we call the force, the *weight* of the object.[1] This force is constant for any two given bodies, at any given distance from each other, at any part of space (at least within the solar system); but depends upon the bodies and their distance apart, according to Newton's well-known law

[1] More strictly, the action of gravitation between an object and the earth is called the *force of gravity*, or merely the *gravity* of the object. *Weight* is the force of gravity, diminished by the effect of the earth's rotation, the object being supposed to rotate with the earth. In other words, weight is the force of gravity minus the so-called centrifugal force arising from the body's rotation with the earth.

that the force is proportional to the product of the masses, and inversely as the square of the distance between the centers of the bodies.

Gravitation and Heat. — Suppose some heavy "weights" to be suspended by a cord passing over pulleys and wound around a vertical drum as in Joule's experiment (see any text-book on physics). Suppose this drum to turn on a vertical shaft which extends downward into a closed cylindrical box containing water, and to have paddles projecting laterally from it inside the box. Suppose the box to be securely held to prevent turning and the weights to be allowed to descend; the water will be violently stirred; gravitation energy is imparted to the weights; they descend and through them the gravitation energy is transformed into energy of elasticity in the cord and as such transferred through the cord, the shaft, and the paddles, to the water. It is there transformed into kinetic energy of eddies in the water, which is in turn transformed into heat through the friction of the water upon itself and upon the paddles and the walls of the cylinder. All losses being allowed for, the gravitation energy expended (weight × distance of descent) is the equivalent of the heat produced (mass of water × number of degrees of the rise of temperature, with allowance for heat capacity of the box).

Kinetic Energy into Heat. — The total gravitation energy imparted to the "weights" in the foregoing experiment, during their steady slow descent, is equal to the kinetic energy which they would have acquired in a free fall through the same distance. By a suitable device, the weights might have been allowed to fall freely through the same height as before, and the kinetic energy thus acquired might have been made to drive the paddles in the machine of the foregoing experiment. The heat generated by the total expenditure of this energy would be found to be precisely the same as before.

In a collision between a bullet and a target (taken as the type of the collision of an inelastic substance like lead, with a resisting body), the kinetic energy of the bullet is largely transformed into heat. This arises through the friction of the parts of the bullet upon each other as it becomes distorted, through the friction of the bullet upon the target in penetrating it, and through friction of parts of the material of the target upon each other. In such a case, however, a considerable portion of the energy is changed into other forms than heat. This is also true in most unconstrained transformations.

The transformation of kinetic energy into heat is going on at almost every point in the universe where motion exists; since it takes place wherever motion is in progress against frictional resistance, and friction is almost universal in its occurrence.

In a descending clock weight, gravitation energy is transformed into heat through the friction of the moving parts of the clockwork and by the air currents set up by the pendulum and the moving wheels. Some of it is transformed into energy of sound vibration which, in turn, is broken down into heat energy through the impact of the vibrations upon inelastic substances or from transmission through them.

Electrical Energy into Kinetic Energy. — Two pith balls electrified with positive charges tend to recede from one another; in other words, if free to move they will be accelerated away from each other. Some form of energy, which we call electrical energy, has been brought into action by this process of electrification, and is exerting a force upon each of the balls, tending to separate them. When they are free to move, this electrical energy must, under the principle of conservation, be transformed into the kinetic energy resulting. The charge of electricity on either body does not diminish however far apart the two

may be separated, as can be shown by the ordinary tests for charge; but the power of the charges to produce kinetic energy diminishes with the distance between the balls. The electrical energy can be experimentally shown to diminish in the proportion as kinetic energy is produced.

If the pith balls are not free to move, but are opposed by frictional or other resistance so that they can move with but a limited velocity, the electrical energy is transformed, not into kinetic energy, but into such other forms as heat if the resistance is due to friction, or gravitation if the resistance is due to weight, and so on.

Electrical Energy into Heat. — When a current of electricity is flowing through a conductor, it produces heat in that conductor at a time-rate CV, where C is the current flowing in the conductor and V is the difference of the electrical pressure at the ends of the conductor (difference of potential) causing this current. Heat is generated in proportion to this product CV. Hence under the principle of conservation, this product is the measure of the electrical energy expended. In just what this energy consists we do not know; but of its existence besides the production of heat, we have evidence in the production of kinetic energy, and in the maintenance of motion against resistance, as will be shown in a later paragraph.

Chemical Energy into Heat. — If a piece of carbon be brought into contact with oxygen at a sufficiently high temperature, the familiar phenomenon of combustion occurs, that is, the carbon combines with the oxygen producing carbon dioxide, and the operation is attended by the evolution of heat. Similarly zinc in combining with oxygen evolves heat; and in general the direct chemical combination of two or more elements likewise produces heat. Since heat is produced, that is, a form of energy is made to appear, some energy must have disappeared, and this is the energy which gave to these substances their tendency to

enter into chemical combination. As has been stated, this may conveniently be called chemical energy.

This form of energy is even more obscure as to its nature than gravitation energy, and in a certain sense is more complicated in character; for it apparently exercises a discrimination between portions of matter, which is not dependent merely upon their masses but upon the kinds of substance of which they consist. Thus when carbon and oxygen unite, the ratio between the mass of carbon and the mass of oxygen in the combination is always a definite one. Also the quantity of heat generated per unit mass of carbon entering into combination, or per unit mass of carbon dioxide resulting from the combination, is a definite amount always the same under the same external conditions. Similarly, hydrogen combines with oxygen in definite proportions by mass; hydrogen with chlorine; and so on throughout a long range of direct combinations of elements. In each case a definite quantity of heat is produced in the combination per unit mass, reckoned in terms of either constituent or of the resulting compound. The total quantity of heat is in direct proportion to the mass of the compound.

This heat of combination appears to be the equivalent of all or nearly all of the chemical energy which causes the union, unless some change such as increase or diminution of volume or the production of electrical current, attends the direct combination. In such instances the sum of all the sundry quantities of energy produced is equal to the heat which would be produced by the same combination if heat alone resulted. The heat produced, with due allowance for such external actions as attend expansion, etc., is taken therefore as a measure of the chemical energy which is brought into play in the combination.

Speculations as to the nature of chemical energy and the selective properties of the elements have been but scantily successful with the remarkable exception of Professor

J. J. Thomson's development of the Vortex-atom Theory (Part II., Chap. IV.). Some further, though crude, suggestions are briefly given in Chapter V. of Part II.

The above instances are taken as merely the simplest ones typical of this conversion of energy. The reactions between chemical compounds involve, in addition to simple transformation of chemical energy into heat, other transformations more or less complex, which render them less well suited for elementary illustration than the direct combination of elements.

Chemical Energy into Electrical Energy. — The energy which in the foregoing section was shown to be transformable into heat, is under suitable conditions convertible directly into electrical energy. Perhaps the most familiar illustration is that of the simple zinc-copper-sulphuric acid cell. This consists of a jar containing dilute solution of sulphuric acid, into which, and not in contact with each other, dip a piece of zinc and a piece of copper. If these two metals be connected by wire outside of the jar, a current of electricity will flow in the wire and through the cell, and will produce heat (although at a very slight rate since the amount of energy expended in the wire is small). The chemical action going on in the cell may be regarded as merely the oxidation of zinc, although the operation is attended with the evolution of hydrogen gas together with the production and solution of zinc sulphate. If the total electrical energy be measured in the usual way by measuring the total current and electrical pressure in the circuit, this will be found an exact equivalent for the heat which would be produced by the direct oxidation of zinc, with due allowance for the minor transformations attending the solution of the zinc sulphate, the evolution of the hydrogen, and the reduction of volume of the oxygen when it combines with zinc in the direct oxidation. Thus, by means of suitable processes, chemical energy may be transformed directly

into electrical energy; and this in turn may be changed directly into heat, or by a motor into mechanical energy or other forms.

Magnetic Energy into Kinetic Energy. — This conversion is familiarly illustrated by the motion of a free piece of iron towards a magnet. Moving under the influence of the magnetic energy of the magnet, the iron acquires increasing amount of kinetic energy. This action of the magnet is so closely related to the action which accompanies a solenoid carrying a current of electricity, that an intimate relation between electrical and magnetic energy is at once suggested, and this has led to an electrical theory of magnetism which regards the two energies as fundamentally the same, magnetic energy being merely a manifestation of electrical energy. The object here is merely to point out its existence.

Heat into Mechanical Energy. — We are familiar with the series of conversions of energy which take place in the ordinary production of mechanical energy from coal through the steam engine. They may be enumerated as follows: — transformation of chemical energy of coal and oxygen into heat energy, when the coal is burned under the boiler; the transformation of some of this heat into other forms of energy in producing steam from water, as well as the storing up of heat energy as such in the steam; of this total energy of the steam, a part is converted into mechanical energy as the steam is expanded behind the piston of the steam engine, driving the piston forward against the resistance of the machinery. In this process, during the expansion of the steam, some heat is transformed into mechanical energy.

A similar illustration consists in enclosing a quantity of compressed gas under a piston loaded with a suitable weight, and allowing the gas by its expansive pressure to force this piston backwards against the resistance afforded by this

weight, thus producing gravitation energy. This will be done at the expense of the heat energy contained in the gas, as will be indicated by a lowering of its temperature.

Neither of these illustrations constitutes anything like a demonstration that heat can be converted into other forms of energy. Such a demonstration of a perfectly rigid character can be given by use of the Carnot cycle and imaginary heat engine. Demonstration may be found in text-books of physics, and is clearly expressed in Tait's "Heat" and Maxwell's "Theory of Heat."

Electrical Energy into Mechanical Energy. — If a steady current of electricity be sent through an electric motor, and the losses in the motor duly allowed for, it will be found that the motor is capable of producing kinetic energy of an amount equivalent to the product CVT, where C is the current (supposed to be constant), V the difference of electrical pressure at the terminals of the motors, and T the duration of the experiment. If the motor is not made to produce kinetic energy directly, it can be made, as it usually is, to maintain motion of machinery against resistance such as friction, weight, etc., thus producing energy of other recognized forms. The quantity of energy thus furnished will also be found to be in direct proportion to CVT. Thus in either case we have the product CVT producing an equivalent of some form of energy, so that it must correspond under the law of conservation to the quantity of electrical energy expended.

We also find that, when the energy produced by the current corresponds in quantity to the heat produced in the wire, the current is no longer capable of producing other energy; or if other energy is being produced simultaneously in proportion to CVT, it is no longer capable of producing this amount of heat. If both heat and mechanical energy are being produced by the current, then the quantity CVT must be proportional to the sum of the energies of the two

kinds produced. These various facts not only confirm the inference that a current of electricity possesses electrical energy, or flows in virtue of electrical energy, but also the inference that the product CVT measures the energy of a steady current.

Remarks on the Transformation of Energy. — The various statements made in this section are necessarily based on hypotheses of one or another kind, and are therefore to be distinctly regarded as suggestive rather than descriptive.

When a free body is moved by gravitation energy, the transformation of that energy into the kinetic energy of the body appears to be a direct one. Gravitation seems to act directly upon the substance of the body without the intervention of any other form of energy; but this appearance may be illusory. Similarly when the kinetic energy of a body is transformed into energy of gravitation, the process is also direct. But whether gravitation acts directly upon the matter of which the body is composed, or whether it acts only upon the aggregates of matter which we call atoms, or upon the more indefinite aggregates which we call particles, is not known. We know gravitation only through its association with bodies. That is, we know it as a property of bodies, and the only hypothesis of any weight respecting its nature locates it as acting upon the atoms from without. Neither of the existing hypotheses (Le Sage; Vortex-atom) indicates that it acts upon the matter of which the atoms are composed, and by the vortex-atom hypothesis the differentiation of matter into atoms is a necessary antecedent of gravitation. In these hypotheses the action of gravitation energy is, however, not direct but through intermediate conversion into energy of elasticity.

The transference of kinetic energy from one elastic body to another in collision, although at first sight seemingly a direct process, really involves a double transformation. All

the energy transferred is transformed, in the process, into energy of elasticity; transferred as energy of elasticity, from particle to particle, and from one body to the other, and is retransformed into kinetic energy in the second body. This may be most simply traced out in imagination in a collision of two perfectly elastic spherical balls of equal mass and of homogeneous structure.

The transfer of heat energy from one body to another, or from particle to particle of the same body, is supposed to involve the same sort of transformation into elastic energy and back again into kinetic, as occurs in the above instance of collision of elastic bodies. This of course is based on the supposition that heat is the kinetic energy of random vibration of the molecules or particles of bodies.

Transformation of heat energy into other forms, through the motion imparted to a body (*e.g.* a piston), by an expanding gas, is likewise double. It involves the transference of the elastic energy of the gas into elastic energy in the body against which it presses, and the transference of this elastic energy from particle to particle of the body and its final transformation into energy of motion of that body, or into such other form as may be the result of the process. The energy of elasticity of the gas is ordinarily ascribed to its heat energy, so that if we go back a step further in the process we have, — first, the transformation of the heat energy into elastic energy, that is, the transformation of kinetic energy of the molecule into elastic energy as the molecule strikes the side of the body acted upon; second, the transference of that elastic energy to the particles of that body, and so on.

This intervention of elasticity, by which other energy is transformed into elastic energy, and thence into some other form, seems to be universally present as an intermediate step in all energy transformations. But this fact is so commonly overlooked as to sometimes render it a surprise

when stated. Indeed, few text-books even name elasticity among the forms of energy. The oversight may be accounted for largely upon the ground that the double transformation usually involves little loss or dispersion of energy, so that the omission introduces no conspicuous error.

The presence of elastic energy and the transformations of it in structures under strain, determine the forces in the members of such structures; and much aid might be rendered to students in the study of statics of structures by the consideration of this energy and its transformations.

Further Remarks on the Definition of Energy. — This ability of energy to transform itself seems at first sight beyond our original definition, but it may not prove to be so. For, if all energy should prove to be in reality kinetic (cf. Part II.), every transformation is but a redistribution of motion among bodies or parts of a body. Thus a transformation of molar kinetic energy into heat may be only a change from molar to molecular kinetic energy. If gravitation energy is due to some mode of motion of special particles of substance as in the Le Sage and Vortex-atom hypotheses, the transformation of this into kinetic energy is again merely a redistribution of kinetic energy. Like remarks would apply to the other forms.

Exertion of Force by Energy. — It may not be superfluous to direct attention again to the fact that the definition of force does not imply that energy is at all times exerting force, or that all forms of energy are capable of exerting force. Whether or not the latter is true, is impossible to determine in the present state of knowledge. Inspecting all possible transformations of energy one after another, it appears upon the surface that all forms of energy do exert force. Looking more closely, however, we find that, in cases where the nature of the action which we call force is best known, the force is exerted only through the intervention of elastic energy. And, in cases where we are obliged

to resort to hypotheses to explain the action, the only acceptable hypotheses assume the existence of elastic energy, which is so called into play as to exert the force. For example, in the transference of kinetic energy through collision of bodies, we might at first sight say that the kinetic energy exerted force, but upon examination, we find that the kinetic energy of the acting body is certainly transformed into elastic energy, which exerts force between the parts of this body, and between this body and the one acted upon. This is the case of transference of energy which we best understand. In this the only force involved, so far as we know, is that exerted by elastic energy. The process is not a direct transference of kinetic energy, but involves an intermediate transformation into and out of energy of elasticity. Again, we say that gravitation energy exerts a force; but, when we attempt to explain how it exerts force, we are obliged to resort to hypothesis, and every explanation of this kind yet offered, involves the action of elasticity and ascribes the exertion of force to this elastic energy.

Accustomed as most students are to regard heat energy as kinetic, heat is perhaps usually thought of as not producing force. But if we analyze this view, we shall see that it results from hypothesis and is not a direct statement of the facts. Suppose any substance whatever, for instance a solid body, to be enclosed by an unyielding boundary. Let heat be imparted. The temperature will in general rise, by which we recognize that a portion at least of the imparted heat remains as heat in the substance. At the higher temperature, the substance will exert a pressure, — a force, — and usually a very great one against the retaining boundary. This force so far as we *know* is due to (is the action of) the heat energy. It is due to that portion of the heat which remains as heat, and not to any portion which has been transformed. If, therefore, we lay aside all hypothesis we are compelled by the facts to say that, in this example at

least, heat exerts force. If, on the other hand, we choose to adopt the hypothesis that heat is molecular kinetic energy and that this pressure arises from the bombardment of the boundary by the elastic particles of the substance in their heat vibration, then we transfer the exertion of force from heat energy to elastic energy. Thus, *without* hypothesis, we must assume that heat exerts force; *with* hypothesis we again revert to elastic energy as the ultimate source of force.

While we must not fail to discriminate sharply between propositions deduced directly from observation and those based upon hypothesis; still in view of the apparently universal production of the elastic energy as an intermediate step in all energy transformations, and in the exertion of force, we can hardly avoid surprise at the scant recognition accorded to this form of energy.

Dissipation of Energy. — In stating the principle of conservation, the reservation was made that the principle did not declare that all the energy of one kind could be transformed into energy of other kinds. Of the line of facts to which this relates only the briefest mention need here be made.

In almost all transformations, whether natural or artificial, some portion of the original energy is changed into heat through the overcoming of frictional and other resistance. There is thus a continuous tendency to increase the amount of heat energy of the universe at the expense of other forms, and this tendency is not offset, so far as known, by any reverse process. Moreover in the transformation of heat energy into other forms, there is a continuous loss of availability of heat energy. Thus it is shown to be a natural law that even the most efficient heat engines, taking heat from a source at a given temperature and working between that temperature and a lower one, — as all heat engines must work, — can convert into other forms of energy only a definite and limited fraction of the heat taken up; the remain-

der of the heat energy being given out in the form of heat at the lower temperature. Also it is impossible to derive any power from heat by cooling a body below the coolest of surrounding objects.

Again in every heat transformation there is a greater or less accidental escape of heat; further there is a continual flow of heat by conduction, radiation, and convection, from sources at a higher temperature to points at a lower temperature.

Thus it is shown that there is first, a continuous transformation of other forms into heat; and second, a continuous lessening of the availability of heat by the approach of all bodies to a common level of temperature. Should such a condition ultimately be reached, there is no recognized way by which this mass of unavailable heat could be retransformed into the other forms of energy. Thus it appears that the universe may be approaching an ultimate condition very different from the present in respect to the diffusion of energy and the consequent condition of matter.

This progressive change in energy is called the dissipation or degradation of energy. That there may be in nature some compensating process, either continuous or saltatory, is not denied, and speculations upon this topic are possessed of great interest.

CHAPTER VIII.

FORCE MEASUREMENT.

Quantity of Force. — The amount of a force may be practically measured in various ways, but amount or quantity of force must be first defined in a mathematical way consistent with the definition of force itself. Since force is "that action of energy by which it produces tendency to change in state of motion of bodies," since this change involves the acceleration of the body if free and therefore the imparting of energy to it as referred to its former state of motion, force is naturally measured as the rate at which energy is thus imparted. Now it is a matter of observation that when a body moves under this action of energy, it receives energy at a definite space-rate at every point of its path, independently of the epoch or the velocity at which it traverses that path, except in special cases. That is, if we fix our attention upon any given point of the path, then the amount of energy which the body will there receive in traversing any specified distance along the path will always be the same whether the motion occurs at one time or another, and whether the velocity of the body at the time is great or small.

If the action be uniform, the force is called constant, and the space-rate of transfer of energy to the body will be the same at all points of the path. Let E represent the total energy imparted to the body in traversing a distance S along the path, then the space-rate will be E/S, and the force will have the constant value

$$F = E/S.$$

In such a case, to measure F we might measure E and S. Their quotient would give the value of F. If the action is not uniform, that is, if it is not the same at all points of the path; and if as before, E represents the total quantity of energy received by the body in traversing the total distance S; then E/S will represent the *average* space-rate, and F will represent the average value of the force. The force in this case is said to be a varying force,—varying with respect to the value of the force along the path and not to the time of its action. In the case of a varying force, if e represents the quantity of energy received by the body in traversing a very small length s of the path, then the average value of the force over this small element s will be $f = e/s$. As s is taken successively smaller and smaller, the limit toward which the ratio e/s approaches is more and more nearly the actual value of the force f at the point of the path where s is taken; and this limiting value (denoted in the calculus by de/ds) is the actual value of f at that point.

A primary method for the measurement of force may, therefore, consist in measuring the space-rate of transference of energy when this force is exerted.

When a body moves freely under a force, it moves with accelerated motion. If the force be constant, the motion is uniformly accelerated. It is consistent with the definition of force to measure it in terms of this acceleration and of the kinergety (or mass) of the body accelerated. Force may be thus measured by the product MA, where M is the kinergety or mass of the body, and A is the rate of acceleration at any given instant, or point of the path. This method of measurement will be discussed in a later paragraph. It is in some respects less satisfactory, and is less simple than the measurement as a space-rate of energy. The latter method connects force directly with energy, and brings into account the fact that this action of energy is connected

directly with space and not with time. It has the special advantage of applying to all motion whatever, whether free or constrained; whereas the time-rate measurement applies only to freely moving bodies.

To ascertain the average value of a force F, we have then to measure the energy E transferred to the body while moving over the distance S. If the energy transferred appears wholly in the kinetic form, as it does when the body has entire freedom of motion, then $E = \frac{1}{2} MV^2$; so that E becomes known, if we know the mass M of the body and its velocity V acquired while moving freely under the force through the space S, starting from rest. Thus using the centimeter, gramme, and second, as units of length, mass, and time, respectively (C.G.S. system), suppose a body having a mass of 5 grammes (= M) to start from rest and to move freely through a distance of 200 cm. (= S) under the influence of a force F whose average amount it is desired to measure. Let its velocity at the end of this space be 40 cm. per second (= V). Then $E = \frac{1}{2} MV^2 = \frac{1}{2} \times 5 \times 40^2 = 4000$ ergs. The average force therefore is $F = \frac{E}{S} = \frac{4000}{200} = 20$ dynes.

If the motion is not free, then some of the imparted energy does not appear in the kinetic form, but is transformed into one or several other forms. If the amounts of each of these be measured (by methods to be indicated), their sum total will (under the principle of the conservation of energy) be the total energy E transferred; and thence F may be computed. The descending clock weight is an instance of this case.

Practical Measurement of Forces. — Forces may be measured by other phenomena which accompany them as well as by the energy transferred. The resulting methods will of course be secondary, that is, before being adopted they must be shown to yield results consistent with the primary (energy) method.

The most general of these consist in opposing to the unknown force a known force just sufficient to balance it. When no motion results, we know that the two forces are equal and opposite, — a condition known as equilibrium of the forces. This depends on the proposition that when two equal and opposite forces are applied at the same point of a free body, no motion ensues, — a proposition which may be demonstrated as follows. If a body move under the application of a given force, it will acquire energy at a definite space-rate, and will therefore move over a definite space in a given time, in the direction of the force. Suppose such a force to be exerted for a short time and to be followed by an equal and opposite force for an equal time, the body will move a definite distance under the first force and an equal distance in the opposite direction under the second force; so that, at the end of the second period, it will find itself at the original starting-point. Suppose a succession of such operations to occur; then the shorter the periods of each, the more the action will resemble simultaneous application of two equal and opposite forces. Also the excursion of the body acted upon will become smaller and smaller, so that as the periods become shorter and shorter we approach a condition of simultaneous forces and no motion.

In the actual measurement of forces, or in an application of known forces, use is made of this balancing of the effects attending equal forces. The instruments and devices are numerous, but they may be grouped for the most part as applications of the principle of the lever, or of elasticity (largely by use of the coiled spring). These methods imply the ability to produce known forces (cf. p. 101), and in this respect are to be distinguished from the energy method of measuring force (p. 97). As typical of them the spring balance will be briefly described.

Spring Balance. — If we suspend some free object from

the end of a coiled spring (taken merely as the type of all elastic bodies), it will descend under the influence of its weight, stretching the spring, and will finally come to rest with the spring stretched so that its elastic force is just equal and opposite to the weight of the body. The spring will always show the same stretch (strain) when the same object is hung upon it at the same point of the earth's surface. But we know that under this condition, the weight of the object is always the same (cf. p. 102). Hence whenever we see the spring stretched to this amount and held in equilibrium, we know that a force equal to that of the spring, and therefore to the weight of the above object, is exerted against the spring. It is of course assumed that minor disturbing causes are eliminated. By attaching an index to the spring and applying sundry weights, we may graduate the instrument and thus have a spring-balance or spring-dynamometer. Similarly a continued distortion of any elastic body indicates not only a force in the body but one against it. If motion is taking place, the two forces are unequal when the state of motion is changing, equal when the motion is uniform.

This operation of graduating the spring-balance implies the ability to construct previously a graded set of bodies which shall have known relative weights. The explanation of how this is possible involves a most important practical proposition in the system of physical measurement, and this will be detailed in the following section.

Standards of Force. — As already stated, it is requisite, in certain methods of measuring force and in the application of known forces, to have at command the means of producing, at any time or place, accurately known forces whose amounts are under control. For this purpose, we practically employ the weights of certain bodies. These bodies consist of suitably shaped pieces of metal or other durable substance, and are for convenience arranged in sets, each piece

having a weight which is a multiple or sub-multiple of the weight of a standard piece of metal which is carefully preserved. The weight of this standard piece of metal, at a standard locality, affords us our practical standard for the measurement of weight, and of force in general. The two recognized standard bodies are (p. 63) the "International Kilogramme" and the "Imperial Pound Avoirdupois." It will be shown later (p. 111) how this graded set of bodies, whose weight furnishes us with a means of determining relative weights, also serves, through the assumed relation between mass and weight, and through the definition of *weightal* (p. 10), as a graded *set of masses* or weights for use in the measurement of mass or weightal. The use of this set of bodies in the primary measurement of kinergety has been described in Chapter VI. It may here be again noted that the standard body which serves to provide the standard of force also serves as the standard of kinergety, of mass, and of weightal.

The employment of the weight of a standard body as a standard of force (not necessarily as a *unit* of force), depends upon several considerations, namely: —

I. The constant intensity of weight (p. 82) at any given point of the earth's surface. By "intensity of weight" is meant the weight of the standard body.

II. The uniformity of weight throughout a space large enough for experimental purposes (*e.g.* throughout an ordinary room).

III. An accurate knowledge of the relative intensity of weight at desired points upon the earth's surface.

All three of these points are capable of investigation by two methods directly or indirectly. These are, the rate of acceleration of a freely falling body, and the period of vibration of an "invariable" pendulum. The law of universal gravitation, and the constancy of this force between members of the solar system, affords us corroborative evidence

of the highest value as to the constancy of terrestrial gravitation, except as this may be modified by local variations.

The constancy and uniformity of weight, or more properly of the intensity of weight, at a given point and throughout a given space might be deduced from the observed rate of acceleration of a given freely falling body experimented upon at different epochs and at different portions of the space under consideration. This direct method, however, is deficient in accuracy.

Observations with the "invariable" pendulum are those upon which we chiefly rely. But all such direct measurements are of course confirmed by the absence of those contradictions which would almost certainly arise in an immense multitude of ordinary observations and phenomena if weight were not constant at a given point.

The invariable pendulum consists of a metallic pendulum so constructed that its kinergety (and therefore mass) and its dimensions remain constant, and that its "length" (*i.e.* the length of an equivalent simple pendulum) is unalterable except through temperature, and so on. Such a pendulum when swinging on knife-edges at a given point of the earth's surface, has a time or period of vibration which depends upon the kinergety and dimensions of the pendulum, and upon the intensity of weight at that point. But as the kinergety and dimensions are invariable, the time varies only with the intensity of weight, and is inversely proportional to the square root of that force. The intensity of weight is proportional to the rate of acceleration (usually denoted by g) which it will produce upon a body falling freely in vacuum. The results of pendulum observations are, for convenience, usually expressed by giving the value of g, deduced from them for the place of observation, or the relative value of g at that place as referred to that at some standard point.

Through observations with such an invariable pendulum or through other analogous pendulum researches, it has been shown that the intensity of weight is constant from year to year. If this were not so, clocks would require a continual modification of the lengths of the pendulum to correspond with the variations in weight, a fact which would become very evident in astronomical observatories even if the variations were very small. Observations with invariable pendulums at neighboring points show that the variations in the same horizontal plane throughout a space such as an ordinary room must be exceedingly small. Observations at many and widely distributed points over the earth's surface have given the relative value of g at these points with high accuracy, and from them the following general equation has been deduced for expressing the relative value g of the acceleration of weight at any point whose latitude is λ and whose altitude above the sea-level is H (expressed in meters),

$$g = g_{45,0}(1 - 0.00\,259\cos 2\lambda - 0.00\,000\,020\ \mathrm{H}),$$

$$g_{45,0} = 980.6\frac{\text{cm.}}{\text{sec.}^2}\ \text{approx.}$$

This is the approximate average value of g at latitude 45°, sea-level. Different experimental values of $g_{45,0}$ vary widely in the next place of figures.[1]

From experimental observations with a simple pendulum, the absolute value of g at any place may be determined with an accuracy of about one part in one hundred thousand. The relative determination at two different places by means of the invariable pendulum may be made with at least ten times this accuracy. The value computed for any particular place from the above formula would be liable to an error exceeding one part in one hundred thousand. These statements are intended to be only roughly approximate.

[1] Very recent observations render it probable that near the earth's surface the coefficient of H is more nearly 0.00 000 030.

The total variation throughout the volume of an ordinary room would probably not exceed one part in one million. The constancy of the weight of a body from one time to another is undoubtedly far greater than the accuracy with which it can be tested.

We may now proceed to show how a set of bodies which shall furnish us with a graded set of weights may be constructed by the aid of the equal-arm balance. The accuracy of the relative adjustment of these is limited only by the sensitiveness of the balance, which is between one part in one million and one part in ten million. Thus if we could use this set of weights in connection with an equal-arm balance we could obtain forces at any given point of the earth's surface which would be relatively exact to the limit just stated. If, however, we make use of this set of bodies at any other point, they afford us forces whose absolute amount is different according to the value of g at the place. This value may vary by as much as one part in two hundred at extreme points of the earth's surface, but between any places where such work is likely to be done, the extreme variation would not exceed one part in four hundred. Thus to assume that the weights of this set are constant at all points of the earth's surface is to introduce a possible error of one part in two hundred; while applying a correction for the relative value of g will enable us to employ them at any desired place with an accuracy of about one part in a hundred thousand if the above formula is used, or with a higher accuracy if a direct relative measurement of g is made with an invariable pendulum. In using this set of bodies at different places, therefore, for the production of known forces, we must know the relative value of g at the place where the weights are standard and at the place where they are used, with the desired degree of accuracy.

The special application of the equal-arm balance in the construction of the bodies having the graded set of weights

is as follows. It is the process called "weighing by taring" and introduces no principle not thus far discussed in this book. Suppose that we place in one pan of the equal-arm balance the standard body, and produce equilibrium by placing in the other pan sand or any convenient material (a preliminary set of weights for instance). We thus evidently exert through the beam of the balance at the knife-edge carrying the standard body, an upward force equal to the weight of that body. If now without disturbing the counterpoising material, we should replace the standard body by any other object, for instance the body whose weight we desire to make equal to that of the standard, there will not be equilibrium unless the weight of this object is exactly equal to that of the standard body. We may then add to, or take away from, the material of the object until, after repeated trials, it is found that equilibrium occurs. The weight of the object is then precisely equal to that of the standard body (within one part in one million or less with suitable precautions and corrections).

Having thus primarily established two bodies having equal weights, we may make a third which shall have a weight equal to the sum of the two, and proceeding thus we may build up a set of bodies whose weights shall be in any desired gradation. This may be called a graded "set of weights," although this term must be understood as qualified by the variation of weight with locality. Assuming for reasons stated in Chapter XI. that mass is proportional to weight, this graded set of bodies may be and generally is called a *set of masses*. It would be better called a graded *set of weightals* (p. 153).

Unit of Force. — In choosing a unit of any kind whatever, we are governed solely by considerations of convenience. In framing a system of units for the measurement of quantities of all sorts, it is of great advantage that the selection should be systematic and not entirely arbitrary. By a suit-

able selection, it is practicable to avoid numerous inconvenient mathematical constants in fundamental equations connecting quantities of various kinds. This will now be illustrated in the case of the unit of force.

Since force is measured by the space-rate of transference of energy, the unit force will be such a force that under its application, energy will be transferred at a unit space-rate. If the force is constant, then under it a unit quantity of energy will be transferred, when its point of application is displaced through unit distance in the direction of its action. Either of these statements constitutes a general definition of the unit of force. We have, then, by the proper choice of units

$$E = FS$$

without the introduction of numerical constants.

Next, inasmuch as the expression $\frac{1}{2}$ MV^2 has been universally accepted as the expression for kinetic energy, we must have

$$FS = \tfrac{1}{2} MV^2$$

where F equals the force (average or uniform) which, moving a free body of kinergety M, through a distance S, from rest, produces in it a velocity V.

If a uniform force F acts upon this free body, the motion will be uniformly accelerated. Let A represent the rate of acceleration produced; then by the law of uniformly accelerated motion, the velocity V, after the force has been exerted through a distance S, will be such that,

$$V^2 = 2\,AS.$$

Substituting this in the above expression we have,

$$FS = \tfrac{1}{2} M \cdot 2\,AS, \text{ or } F = MA.$$

The force F is therefore A times the number, M, of units of kinergety in the body. This expression is general for all forces, A being the observed rate of acceleration produced

on any kinergety M. The force under which a unit acceleration would be produced on a free unit of kinergety would then be a unit of force, since if $A = 1$, and $M = 1$, F will be equal to one. We may, therefore, again define the unit of force as that uniform force which will produce a unit rate of acceleration in a unit kinergety. This is also a general definition, and is a more convenient one to employ in connecting the unit of force with the standard of force.

Let us now apply it to this purpose. We know that any kinergety, and therefore the unit of kinergety, falling freely under its own weight at a point where the acceleration due to weight is g (see p. 103), will be accelerated at that rate g. Therefore, as the force is proportional to the rate of acceleration, the weight of a unit of kinergety must be g times the unit of force. The weight of one gramme, at any place, must, therefore, be g units of force, or in general: —

The unit of force, in any system, must be $\frac{1}{g}$ of the weight of a unit kinergety, where g is the acceleration due to weight at the particular place.

We have now to connect this definition with a specific system of units and with the standard of force.

The system of units in general use for scientific work is called the "Centimeter-gramme-second System" (abbreviated c.g.s.). In establishing this system the units of length, kinergety, and time are first selected, and are respectively the centimeter, gramme, and second. The unit of velocity then becomes one centimeter per second. This unit of velocity has no specific name. The unit of force which is called the dyne is the force under which energy would be transferred at a space-rate of one unit (one erg) per centimeter, or under which a kinergety of one gramme would receive an acceleration of one centimeter per second. By the above argument, the weight of one gramme at a place where the acceleration of weight is g, would be g dynes.

In other words, the unit of force, the dyne, is one *g*th part of the weight of one gramme. The value of g at the sea-level, in latitude 45°, is about 980.6 centimeters per second. For other localities, if not known more exactly from pendulum observations, it may be computed from the formula on page 103. As the gramme is the thousandth part of the kilogramme, we have thus connected the c.g.s. unit of force with the weight of the kilogramme, that is, with the standard of force. The unit quantity of energy is the *erg*, which is the amount of energy transferred under a force of one dyne, when its point of application is displaced through a centimeter in the direction of the action.[1]

It is essential to clearness to remember always that the words gramme, kilogramme, etc., in the c.g.s. system, denote a kinergety, weightal, or mass and not a force. Whenever force, or the particular force called weight, are referred to quantitatively, the term *dyne* is used.

It appears unfortunate that the unit of energy was not originally defined in advance of the unit of force. If the unit of kinetic energy had been defined as the kinetic energy of a unit kinergety when moving with a unit velocity, the expression for kinetic energy would have been MV^2 instead of $\frac{1}{2} MV^2$, which is now universally employed. We should then have deduced the expression for force from the equation $FS = MV^2$, and should have arrived at the result $F = 2\ MA$ instead of $F = MA$. The weight of the unit kinergety would then have been $2\ g$ units of force, and the dyne would have been $\frac{1}{2}$ as great as at present. The introduction of the fraction $\frac{1}{2}$ in the expression for kinetic energy, was rendered necessary by the fact that the unit of force was defined by the expression $F = MA$ in advance of the definition of the unit of kinetic energy. The expression

[1] For a discussion of British and metric engineering units, see "Appleton's School Physics."

$\frac{1}{2}MV^2$ is now so firmly established by custom, however, that it will doubtless be permanent.

Weight Proportional to Kinergety. — That the relative weights of bodies at any given locality are directly proportional to their relative kinergeties is not axiomatic, but must be proved experimentally. It is most simply demonstrated by the fact that any two bodies of whatever weight, starting from rest, will fall towards the earth with equal accelerations provided that the retardation of air and other resistances are removed. The relation follows from this observation by the following argument. The force under which a body descends, in the experiment, is its weight.

The energy received by each body in descending through the same vertical distance S, starting from rest, is its weight multiplied into the distance S, but the kinetic energy of the two bodies after freely traversing this space S will be in direct proportion to their kinergeties, K_1, K_2, since their velocities are experimentally shown to be equal, but the kinetic energy must be in each case equal to the product WS for that body. That is, for the first body $E_1 = W_1S$ and for the second body $E_2 = W_2\,S$. Now S is, by supposition, the same in both cases; so that,

$$E_1 : E_2 :: W_1 : W_2.$$

But, as already stated, $E_1 : E_2 :: K_1 : K_2$.

Therefore, $\quad W_1 : W_2 :: K_1 : K_2;$

that is, the weights are in direct proportion to the kinergeties, which was to be proved.

This experiment cannot be carried out with the requisite accuracy, but its principle can be indirectly applied by processes far more delicate. Of these the experimental process on which we chiefly rely is the time of swing of pendulums. Observations may be made with pendulums of equal length consisting of heavy masses suspended by a

fine wire. The periods of swing of such pendulums will be found the same, independent of great diversity of material employed in the construction of the separate pendulums, and consequently in spite of great diversity of kinergety of the pendulum. Newton devised and executed the first experimental proof of this character. A more satisfactory, because more accurate, proof comes from the observed time of vibration of pendulums of different forms, where the relative time of vibration is calculated from the form and kinergety of the pendulum, under the assumption that kinergety is proportional to weight. No contradiction between computed and observed times of such pendulums is found, exceeding the limits of error of experimenting. A further confirmation, and a still more general one, arises from terrestrial and celestial mechanics. The elliptical forms of the orbits of the planets, the relation between their distances from the sun and their periods of revolution about it, and their mutual influences upon each other and their satellites, are all strictly consistent with the Newtonian law of gravitation which asserts that the force of gravitation is proportional to the masses[1] of the bodies. Of this force, terrestrial gravitation or weight is a special case; moreover if the force of gravitation were not proportional to kinergety, then, in every planet, stresses of great magnitude would be developed by the action of gravitation towards the sun. Of this, evidence would certainly be given by disruption or distortion of these bodies. No trace of any such effect is perceptible. Further, if gravitation were not proportional to kinergety, free bodies of different kinergety at the earth's surface would not retain their relative positions, and a set of phenomena would arise different from anything which we now observe. This last consideration gives *direct* evidence that weight is proportional to kinergety, while the confirmation

[1] On the assumption elsewhere stated that mass is proportional to kinergety.

of Newton's general law applies to weight only through the assumption that this law holds with equal truth (so far as it relates to mass) for bodies near each other, a proposition whose reliability we have no reason to doubt in the case of bodies near the earth's surface.

By the direct pendulum experiments, the proportionality of weight to kinergety is established with an accuracy of at least one part in a hundred thousand, while the confirmations of the Newtonian law give to the proportion a far higher probability.

From the proposition just established, it follows that a set of bodies whose weights at a given point are graded multiples or sub-multiples (p. 101) of that of the standard body affords us a set of graded kinergeties which are multiples or sub-multiples of the kinergety of the standard body. The relative *weights* of these bodies are reliable within the limit of one part in one million or better (the limit of the equal-arm balance), and the relative *kinergeties* are with little doubt equally reliable. Thus we may assume, with a very high degree of probability, that measurements of relative kinergety by the equal-arm balance are reliable up to the limit of accuracy of that instrument.

Whereas the actual weights of these graded bodies (as expressed in dynes or other unit similarly defined) vary widely according to locality, the kinergeties are absolutely constant, the same at all times and at all places, except when the amount of substance in the bodies is changed through accidental injury.

Weightal.—The quantity of any substance measured off by "weighing" with the equal-arm balance against a standard "set of weights" will be the same at whatever locality the process is performed, although the weight of the portion will vary. When we have to do with this quantity rather than with force or weight, it will be called a *weightal*. This term will be further discussed in Chapter XI. (cf. p. 152).

CHAPTER IX.

WORK.

Nature of Work. — Let the term *source* or *stock* of energy be used to denote some portion of energy of any form, the action of which we are considering. Its location may or may not be known, and it may or may not be associated with some known substance. Inference from observation shows that when a source of energy is exerting against a body the action called force, and when the body undergoes displacement in the direction of the force, then the source gives up energy to the body at a definite space-rate. The direction of the force has been defined as being the direction of the motion of a free body under the force. Whatever the path of the body in any case, the displacement in a given direction is merely the change of position measured in that direction; or in other words, it is the component in that direction when the actual motion is resolved into two components parallel and perpendicular respectively to that direction. The point of application and the line of a force are the point at which the force is applied and a line through that point in the direction of the force. If then a body on which a source of energy is exerting a constant force F, undergoes without rotation a displacement S in the direction of the force, it will receive from the source a quantity of energy E equal to the product of that space-rate into the displacement. But as the space-rate is the measure of and is numerically equal to the force F, the energy will be $E = FS$. This transfer is entirely independent of any other actions which may be in progress at the same time, and is

wholly independent of the path, except that this must have the specified component S. If the force is not uniform, it may be dealt with by the method (p. 99) of subdivision into small steps. No new principle is involved in such a case. This process of transfer of energy by the displacement of a body against which force is being exerted is one case of the *performance of work.* The quantity of work done W is equal to the quantity of energy given up by the source, in this case, so that $W=FS$.

The work is here said to have been *done by* the acting energy, because the displacement is in the direction of its action, and also because the source has expended some of its energy.

If the displacement S of the point of application of the force, in the foregoing instance, had been in the opposite direction, then the source of energy exerting the force F would have acquired an amount of energy $E=FS$, precisely the same as that which it expended in the other case; and this added energy would have been in the form of which this source consists. These statements are also direct inferences from observation. The action in this case is also called the *performance of work.* The amount of work is again $W=FS$, but is now said to be done *against* the energy source under consideration.

From the statements just made, together with the principle of conservation of energy, the following propositions may be deduced. Suppose several uniform forces to be simultaneously exerted in different directions at a single point of a free body at rest, in a manner to produce no rotation. The body will move with uniform acceleration in a straight line. The direction of motion will be such that at the end of any given time the body's position will be the same as though the several forces had been successively exerted, each for this same time-interval. Let F_1, F_2, F_3, etc., denote the respective forces exerted by the correspond-

ing stocks 1, 2, 3, etc., of energy, which may be of different forms. Let S be the actual displacement of the point of application along the path, and S_1, S_2, S_3, etc., the corresponding displacements in the direction of the forces; that is let S_1 be the component of S in the direction of F_1, S_2 the component in the direction of F_2, and so on. Then the amount of energy expended by or imparted to the stock 1 will be F_1S_1. It is then said that work has been performed by or against stock 1 to the amount $W_1=F_1S_1$. Like statements hold for the other forces. Energy is expended *by* the stock, that is work is done by it, when there is displacement in the direction of its force. The work is then accounted positive. If there is displacement in the direction opposite to that of a force, say F_2, the corresponding stock gains in energy of its own form, and work is said to be done *against* the stock in amount $W_2=F_2S_2$, and the work is accounted negative. By the principle of conservation, the quantity of energy given up in this operation must be equal to the total quantity accumulated. But the change in kinetic energy of the body moved, constitutes in this case a part of the accumulated energy. Hence the kinetic energy of the body must be equal to the difference between the positive and negative work done by the several stocks of energy. This is called the resultant work done upon the body. We may then in other words say that the resultant work done upon the body is the algebraic sum of the work done by the several acting stocks of energy, and that this appears in the change in kinetic energy of the body. This change may be either positive or negative. This proposition may readily be shown to be general, that is, applicable to all cases of motion of a body, whatever its initial state of motion, and whether the forces are constant or variable. If the forces are variable, the demonstration proceeds by the method indicated at page 113. The operation may be regarded as taking place in a succession of equal steps, each so short

that the forces may with but slight error be assumed constant. We may then consider what the operation tends to become as these steps are taken shorter and shorter. In this way, the divergence of the imaginary, step-by-step operation from the true one may be eliminated, and the nature of the latter may be discovered. This process is the basis of the calculus method. The kinetic energy dealt with in this case is exclusively that of translatory motion. If the point of application of the forces be such that rotation ensues, the changes in the amount of kinetic energy of rotation must be taken into account. This is also true if the forces are not all applied at the same point of the body. The general case is where each force has any direction and point of application whatever, and the body is in any mode of motion. The work done under or against each force is then the product of that force into the displacement of its own point of application in the direction of the force; and the resultant change in kinetic energy will be the sum of the changes in kinetic energy of translation and rotation.

As a further illustration, take the case of a free body moving in any direction with any velocity, and exposed after passing a given point, to a set of continuous, simultaneous, varying forces so applied that no rotation occurs. The path of the body is determinable by the simple rule that the position of the body at the end of any given time-interval would have been that at which it would have arrived had it traversed in succession the paths which would have been passed over under the separate occurrence of its own motion and of each of the stated forces. This rule is a direct result of observation, and has already been employed in less general form. The change in kinetic energy of this body can be determined under the general rule given above, as may also the various amounts of work done and their algebraic signs. The motion of the body will be accelerated or retarded according to whether the resultant

work done under the forces, that is, the algebraic sum of the separate quantities of work, is positive or negative.

If two simultaneous, constant forces having different directions be applied to a free body at rest in a manner to produce no rotation, the body will acquire uniformly accelerated motion in a straight line. The direction of motion and the space-rate at which kinetic energy will be acquired may be determined as above shown. This rate will be constant, and the actual motion will be therefore the same as though the body were moving under a single constant force having the direction of this motion and applied at the same point as the actual forces. Thus so far as the change of motion of the body is concerned, and also the work done, the two forces might be replaced by a single one as just described. Such a force is called a *resultant* or *resultant force.* If the body is in motion at the outset, the resultant force is not thereby affected but is found as before. If there are more than two forces, the resultant is found by the same procedure. If the forces are variable in direction and magnitude, the resultant has a specific value for each point of the path, determined by the above method. In all cases of several simultaneous forces at one point of a body, the motion and work may thus be computed precisely as if the body were under a single force equal to the resultant, similarly applied, and due to one stock of energy. Whether or not some of the energy exerting the original forces is transformed into some other form within the body, thus producing a stock which actually exerts a force equal to the resultant (and perhaps simultaneously produces other forces), is a point to be studied for each individual case. In cases where elasticity is brought into action, this seems to occur. Such a point of view is helpful in dealing with stress in liquids and solids.

In the discussion thus far given, the term *work* has been applied to the following three processes: The production

of kinetic energy in a free body under a force; the maintenance of the motion of a body against a resistance by its own kinetic energy; the production or maintenance of motion of a body by energy other than kinetic against resistance.

So far then as we have yet proceeded, the significance attached to the term *work* might be expressed by the definition: —

Work is that action of energy by which it produces motion in a free body, or produces or maintains the motion of a body against resisting force.

As thus regarded, work is that process of transformation or of transference of energy which is characterized by the displacement of a body under or against force. This statement might be framed into a definition which should be more explicit than the other. Either of them covers the original and still common scope of the term. But there are other transformations of energy in referring to which the word work is now familiarly used to indicate both the process and the quantity of energy transformed, although neither force, nor motion of bodies, is discoverable. Examples of such are, — the heating of a wire by an electric current; electromagnetic induction; the production of electric currents by heat at the junction of dissimilar metals; the production of heat by radiant energy. Although some theories indicate the presence of both force and motion of a material substance in some or all such cases, their classification as instances of work would not thereby be rendered admissible under the above definition. No other convenient name for these processes has been offered, nor does a separate term seem desirable. The fundamental feature of the nature of work, as shown above, is that of a process of exchange of energy. If we retain for the term this denotation alone, dropping the limitations as to motion and force, much will be gained in breadth and usefulness,

and nothing of definiteness lost. The final definition then becomes: —

WORK IS ANY PROCESS OF TRANSFERENCE OR TRANSFORMATION OF ENERGY.

The term is also used by ellipsis to denote quantity of work, a parallel with the quantitative use of the terms *energy, force,* etc. The word transference is employed in the sense of "imparting" from one portion of substance to an adjacent one, not in the sense of transmission through space. Consistently with this definition, the *performance of work* means the *transference* or *transformation of energy.*

Work is thus clearly to be apprehended as a secondary concept, — a process, an action of energy, as force is, but of a different nature. The distinction between work and force has already been sharply drawn at page 41. Although, as will be shown, quantity of work is numerically identical with quantity of energy transformed, still *work is not energy, nor is energy work,* — a distinction whose neglect finds abundant illustration in current technical if not scientific literature.

Work by and upon. — It is consistent with the view here taken to use the phrases, work done by or against energy, under or against a force, upon or against (in the sense of upon) a body. It is convenient, but not thoroughly good usage, to say, the work done by a body; meaning the work done by some energy resident upon or transmitted by the body. It is not permissible to say, work done by or under the action of a force, since the work is done by or under the action of energy.

Quantity of Work. — From the nature and definition of work, it is sufficiently evident that the quantity of energy transformed or transferred in any given operation is identical with the quantity of work performed. By the use of consistent units the two become numerically equal. There are thus two ways in which work done may be meas-

ured. The first, or energy method, consists in measuring the quantity of energy transferred. We may measure either the energy expended or that which is produced. Thus if the whole work in a certain case consists in accelerating the motion of a body, we may measure the kinergety and the initial and final velocities of the body. Thence may be computed the increase of kinetic energy, which will be the work done. Again, if an electric current heats a wire, or if radiant energy heats a suitable body, and if the heat produced is measured and expressed in units of energy, this quantity will be the work done or energy transformed. When the operation involves several transformations, then all the quantities of energy produced must be measured (separately if necessary) and their sum taken. The second method is usually employed when the forces and displacements occurring can be measured. The mode of calculating the work done under each force from the known amount of the force and the displacement of its point of application, has been shown with sufficient fulness for the present purpose, in the first section of this chapter. The sum of all the positive quantities of work thus found, together with any kinetic energy expended, will be the total work done, provided that no other forms of energy have been engaged in the positive side of the action. Conversely, the sum of all the negative quantities of work thus reckoned, together with any kinetic energy produced, will be the total work done, providing that no other energy than that thus measured results from the operation.

Units of Work. — The scientific unit of work in the c.g.s. system is the erg, — the same as the unit of energy. It is the work done when the point of application of a constant force of one dyne is displaced through one centimeter in the direction of the force. Work may be expressed in any other unit in which energy is expressed, *e.g.* foot-pounds, calories, etc.

Equilibrium of Energy. — When several forces are simultaneously exerted at the same point of a body, and no change in state of motion thereby ensues, there is said to be equilibrium of the forces. Whatever the path or mode of motion of a body, the application to it of any set of forces in equilibrium in no way affects that motion. In such a statement, the space-rate which measures each force is understood to be taken with respect to the same point of reference as is the path or motion of the body itself. But if the body is in motion, the common point of application of the group of forces will undergo the same motion, and work will therefore be done under or against each force as already shown. Hence equilibrium of forces is not a sufficient condition for equilibrium of energy, if by the latter phrase be understood no performance of work under force. Two conditions are necessary for no work under force; namely, equilibrium of forces, and no displacement (absence of motion). Thus if a body at rest be under balanced forces, no motion will ensue and no work will be done. If, now, motion be imparted to the body by the brief application of some other unbalanced force, then the motion will indefinitely continue constant in velocity and direction so long as the forces remain balanced, and work will be continuously performed under certain of the forces and against others as above shown.

Examples of Work. — An object thrown upwards from the earth performs work (or more properly its kinetic energy does so) against its weight. The amount of work done in rising through a vertical distance, whatever be the path, is WH, where W is the weight of the object. The body would rise to a height such that WH plus any other quantities of work done (as against the resistance of the air) was equal to the vertical component of the kinetic energy with which the body started. This component is computed from the vertical component of the initial velocity. The horizontal

component of the object's velocity is unaffected by the action of gravitation, because at right angles to it; and is only lessened by the horizontal component of the air resistance, etc.

There are many familiar cases of the continuous performance of work through approximately uniform motion under balanced forces. Indeed the operation of most machines affords illustrations. A railway train at full speed is an example. As the train starts, the traction force is in excess of the train-resistance (chiefly solid and air friction), and the train therefore accumulates kinetic energy by increasing velocity. The resistance increases as the speed increases, so that with a level, straight track and a constant traction force, a balance of forces will ultimately be reached. The speed will then remain constant and work will be done at a constant rate until some change of conditions occurs.

A pendulum clock driven by a "weight" is a somewhat similar case. When the pendulum is at rest there is equilibrium of forces, and the weight cannot start the pendulum. But after the pendulum is set into a sufficiently wide swing to free the escapement, an unbalanced force will be exerted on it during a portion of each swing. Enough energy will thus be imparted to it to maintain its motion against the resistance of the air, etc. During each interval of freedom of the escapement, the energy of the "weight" also advances the other mechanism through one step against its frictional resistances. The transformations of energy in more detail are as follows.

The case is one where a body moves against resistance and where the acting energy is changed into another form, and in this form is transmitted to the desired locality and there again transformed in accomplishing the result sought. The clock-weight receives gravitation energy at a definite space-rate. The weight, as it descends, moves against the resistance of the suspending cord. It first stretches the

cord, thus transforming gravitation energy into energy of elasticity in the cord until the elastic force in the cord exactly equals the weight (assuming uniform motion). After this, as the weight continues to descend, keeping the clock in operation, the gravitation energy imparted to the weight is transformed into energy of elasticity as fast as the weight descends. This is transmitted from particle to particle through the cord, through the drum on which the cord is wound, and through the mechanism to the sundry parts and surfaces where it is needed to maintain motion against solid friction, the air friction, air resistance, etc. At these parts, namely, bearings, gears, pendulum, and other moving parts, the elastic energy is converted directly into heat, or passes off in such other forms as energy of vibration (of which we have audible evidence), which in its turn is reduced to heat.

Power. — When work is continuously performed, the time-rate of the performance is called *power.* Thus we may have the definition: —

Power is the Time-rate of the Performance of Work, or the Time-rate of the Transference or Transformation of Energy.

If the work is being performed at a uniform rate, then the power is constant, and the total quantity of energy transformed is measured by the product of the power into the time or duration of the performance. If the rate of work is not uniform, then the average value of the power P is $W \div T$, that is, the total amount of work W divided by the time T occupied in its performance. Also the rate of work at any time is the limit of this ratio $W \div T$ as the value of T is taken smaller and smaller approaching zero as a limit. The c.g.s. unit of power is one erg per second.

It is important to discriminate carefully between the use of the terms *power* and *energy.*

The term *power* in technical usage has three distinct meanings. The power of machinery is a term which properly denotes the *rate at which the machine performs work*, and this is ordinarily expressed in the unit called the horse-power, a unit based originally upon a rough approximation to the rate at which a horse is capable of performing work. Power is also used to denote a *force* applied to a machine, as, for instance, in connection with the lever; and it is also used to denote a *class of machines* called mechanical powers.

Strength. — The term *strength* is familiarly employed with a rather indefinite significance. It is used, however, more nearly in the sense of force than of energy or power. That is, we speak of a man who is capable of exerting a great force, such as a powerful push or pull, as being strong, — regardless of whether he may or may not be possessed of endurance. A man would, on the other hand, rarely be referred to as possessing great energy without implying endurance.

Muscular Work. — This is merely the performance of work by muscular energy already referred to at page 22. A certain statement respecting muscular work is often presented as a paradox, although really it is not one. It appears to be true merely because one energy transformation involved in the case is overlooked.

If a heavy object is raised by the hand, work is done upon the object in amount proportional to its weight and to the height through which it is raised. This is done at the expense of muscular energy and is, therefore, muscular work. If, however, the object be held in the hand at a constant height, no work is being done against gravity, that is, no muscular energy is being transformed into gravitation energy; or, in other words, there is no displacement against the resistance of gravity. It is, therefore, frequently asserted that *no muscular work is being done.* This statement is a fallacy. Muscular work *is* being done in maintaining

the muscles of the arm in the tense condition necessary to support the weight. It is true that no work is being done upon the object, but it is not true that no work is being done in the muscles. A transformation of energy, resulting in heat, goes on continuously in the muscles, corresponding to the effort required to maintain their contraction. The assertion in italics is, therefore, simply an inference, which is erroneous because it overlooks this fact.

Methods and Units of Energy Measurement. — The methods by which quantities of energy are measured, and the units in which they are expressed, have been more or less indicated in describing the several forms; for the sake of completeness, however, it may be worth while to summarize them here.

Kinetic energy, as shown at page 106, is measured by the expression $\frac{1}{2}$ MV^2, the factors M and V being separately measured quantities. In cases where it is not convenient to measure M and V, the kinetic energy may be transformed in any convenient way and the resulting quantity of energy measured. Thus, for example, if the body be moving upward freely, the height of its ascent and the amount of its weight would, with a correction for air resistance, give a measure of its kinetic energy, or, if that energy could be wholly expended in the production of heat by the stirring of water, or otherwise, the amount of heat produced would measure the kinetic energy.

Those forms of energy which exert a force which can readily be measured, are measured by the amount of work done under or against this force. That is, the amount of the force is measured as one factor, and the space through which the point of application of the force is displaced in the direction of the force is measured as the other factor. The product of the two gives the work done and the energy expended or received. If the force is not constant, then the average force must be measured, or the expression connecting the force with the path of the point of application

of the force must be known, and the work done must be calculated by means of this. Thus gravitation energy, the energy of electrical and magnetic actions, elasticity, and so on, are measured through the work done under their forces.

The energy of electrical currents is measured through the product CVT, or its equivalent, where these letters denote the current, the potential, and the time respectively. For variable currents, the quantities C and V are the instantaneous values of the current and the potential; and the total energy-production or expenditure during the time T, must be found either by the mathematical law of the change of C and V with the time, or by employing an instrument in which a mechanical action displaces the necessity for the knowledge of this mathematical relation. Again, electrical energy may be measured by the mechanical work which it performs (with corrections for incidental losses). Or again, electrical energy may be wholly transformed into heat and measured as such.

Heat energy is measured by the number of units of mass of water raised one degree in temperature. This arbitrary unit has no simple relation to the units of length, mass, and time, as have the units of the preceding measurements, but is adopted solely because of convenience in manipulation.

Chemical energy, for reasons stated on page 65, is measured by the amount of heat which it produces.

The choice of units in the measurement of quantities of energy is of course determined, as in all other measurements, by custom based largely upon supposed convenience. In scientific work, the unit of energy as measured through work done against or under force, is the *erg*. The unit quantity of heat is the calorie, the amount of heat which will raise one gramme of water from 15° to 16° Centigrade, the temperature being measured by the hydrogen thermometer. This is the equivalent of $4.190 \cdot 10^7$ ergs ["Computation Rules and Logarithms," p. 72, The Macmillan Co.].

CHAPTER X.

POTENTIAL ENERGY.

Definition of Potential Energy.—This term is used to classify all forms of energy which are not assumed to be kinetic. In this sense it has been employed to advantage in mathematical physics for a long time.

The term *potential energy* has, however, a further significance in regard to which much diversity of opinion has already been manifested and still exists. The terms *actual* and *potential* were introduced by Rankine in 1853. As applied to energy, they appear clearly to have been intended to distinguish between energy which a body actually has in possession as kinetic energy, and energy which it has not thus in possession but which is so conditioned with respect to the body as to be acquired by it when free motion of the body is permitted. There was no intention on the part of Rankine, who was an able exponent of the doctrine of conservation of energy, to deny that this potential energy was in existence previous to its acquisition by the body in moving. He merely regarded the energy as *potential to*, or possible to, as distinguished from being *in possession by* the body. This view seems distinctly to locate the energy as outside of the body, or in other words, as acting upon it from without; although this may not have been Rankine's intention. While in the case of gravitation energy this view as to locality may be strongly defended, yet in the case of a bent spring the energy of elasticity, which is equally with gravitation a kind of potential energy, appears almost certainly to be located upon the spring, and very possibly to

be as intimately associated with the particles of the spring as energy of motion is with the particles of a moving body. Electrical and magnetic energy, as well as chemical energy, are naturally classed as potential forms; and reflection will show that while at first sight it may seem obvious that the seat of the energy in these forms is within the individual bodies concerned, yet more careful consideration will either contradict this proposition or render it quite uncertain. At best any decision as to the locality of any of these forms of energy must result from a choice between hypotheses rather than between known facts. Even in the case of heat, which is not classed as potential energy, there is a great difference in the order of certainty attaching to the assertions that heat is a form of energy, and that heat is the kinetic energy of random motion of the particles of bodies. The first of these is as certain as anything in the doctrine of energy. The second has thus far been found incapable of direct proof, although the evidence in its favor is commonly regarded as stronger than for the supposition that any other form of energy is kinetic. (Cf. Part II., Chap. V.) A fair-minded and sufficiently pronounced position upon this subject, therefore, appears to lie in asserting merely that in the case of all the forms of energy which we choose to class as potential, *we do not know* the locality of the energy. To assume on the one hand that potential energy resides upon a body, in the sense of being in full possession by the body concerned, gives the mind at once a bias against any other view; and the same is true if the opposite view is chosen, namely, that potential energy resides outside of, or is not in possession by, the body. In the absence of logical evidence very strongly in favor of one or the other of these views, it is certainly most unscientific to adopt either.

The term *potential energy* has been used largely in a restricted sense in dealing with mechanical energy, and it was at first employed almost exclusively in this sense.

Thus, in a system of bodies the total energy of which depends both upon the configuration of the system and on the motion of its parts, — that is, in which forces existed between the component bodies of the system, and these bodies were changing position, — the total energy was classified as of two kinds. The total energy due to the configuration (or more properly, corresponding to it) was called potential energy; the energy of motion was called actual energy (by Rankine, 1853) or later kinetic energy (by Thomson). Thus the term *potential energy* was introduced by Rankine, "to denote that power of performing work which is due to configuration and not to activity." (For the history of this subject, see "Philosophical Magazine," xxxiii., p. 88.)

It is unfortunate that the phrase, "a body possesses potential energy" and its equivalents are so deeply rooted in scientific literature. It is probably impossible to displace them, so that more than ordinary thoughtfulness needs to be employed in their use.

Examples of Potential Energy. — The significance of this term will be illustrated by a few examples.

When an object is held at some distance above the earth's surface, we know that a force, its weight, is being exerted upon it, tending to move it towards the earth. If it is allowed to descend, it acquires energy at a definite space-rate. The amount of energy which it can acquire before reaching the earth's surface is equal to WS, where W is its weight and S is the vertical distance through which it descends. At its elevated position, it gives no evidence of the possession of this energy (as explained at p. 28), that is, there is nothing whatever in the body to indicate greater energy at its higher position; on the contrary the force upon it, if perceptibly different, is smaller than before. We know, however, that in descending it will receive energy in direct proportion to the vertical distance through

which it moves. However many times the body be carried back to its starting-point and again allowed to descend, it will each time receive the same amount of energy. We are, therefore, justified in saying that this quantity of energy is so conditioned that it is inevitably imparted to the body in its descent. This quantity of energy is, therefore, spoken of as the *potential energy* of the body at a given point with reference to the earth's surface; that is, that quantity of energy is potential, or possible, to the body in this position.

Similarly a bent spring, when released, acquires kinetic energy, or is capable of imparting energy to other objects. In its bent condition, therefore, a definite amount of energy is potential to it, that is, it can and must receive this amount of energy in coming back to its unstrained position. We therefore say that the bent spring has or possesses potential energy, using the term with the same significance as formerly in speaking of the potential energy of an object under gravitation. The energy which is potential to the bent spring, we call energy of elasticity. And it may be worth while to repeat that, by classifying this as potential energy, we do not intend to deny its existence at some locality and in some form when the spring is in a bent position, but merely to imply that it is not possessed by the spring in the sense that a body in motion possesses its kinetic energy, or at least that we do not *know* that it is so.

A piece of iron in the vicinity of a magnet, or of a coil of wire conveying a current, is similarly said to possess potential energy; two conductors conveying a current have potential energy with reference to each other; two substances which have "chemical affinity" for one another, possess potential energy with reference to each other.

In the case of electrified bodies, the potential electrical energy appears at first glance to reside upon the object. An electrical charge, so called, is certainly present upon them, and the presence of such a charge is a necessary

condition of the energy; but when we come to inquire further into the nature of the process, it becomes necessary to assume some intervening medium by which the energy exerts force upon the two bodies. To this medium is ascribed elasticity. The energy of the charges or charged bodies may then possibly reside not wholly in the bodies, but partly or wholly as elastic energy outside of them. This illustrates the above made statement that the choice as to ascribing a locality to the energy is dependent upon hypotheses.

In all the above instances, and in all similar cases which may be cited, energy is potential or possible to the body, that is, the body has or possesses potential energy in virtue jointly of its position with respect to some other body, and of some store or stock of energy which is exerting force upon one or both of the bodies, tending to make them approach or recede from one another. This is true either of the body as a whole, as in the case of gravitation, magnetic and electrical action, and so on, or of the parts of a body relatively to each other, as in the case of the bent spring.

In current phraseology potential energy is sometimes called *energy of position*, since its amount depends upon the relative position of the bodies or parts of the body. But this name is misleading, since the position is merely *one* of the elements in the case, — *one* of the factors of the energy. The body would not possess the potential energy unless it were in the given relative position; but being in that position, neither would it possess the potential energy unless some store or stock of energy were exerting force upon it. In current phraseology also, potential energy is said to be due *to the position of the body and to the action of the force jointly*. This statement, while true if associated with a correct definition of force, often misleads from the lack of such a definition; the production of the energy being then ascribed to the *force* instead of to the *energy* which produces the force.

Any use of the term implying that potential energy is not existent, or ascribing it merely to position, or ascribing it to position and force without distinctly recognizing the relation of force to energy, is productive of confusion. Such use has been so common that the term has fallen into undeserved disrepute. The term is, however, unnecessary and may be avoided without inconvenience, as has been done throughout this book.

Quantity of Potential Energy. — Having in mind the relative character of motion, and therefore of force, we note that when energy exerts a force, it does so upon each of at least two bodies or portions of matter, tending to make them approach or recede from one another, an equal force being exerted upon each. In such a case, an equal amount of energy (from the source producing the force) is potential to each body; or in the more common phraseology, each body possesses an equal amount of potential energy relatively to the other. If both bodies are free to move, each will receive energy at the same space-rate, the force being equal upon both; and they will approach or recede with velocities (relative to their original points of rest or to their mutual center of gravity) such that the accumulated kinetic energy of each is proportional to the distance traversed from its original point of rest. From this it is easy to deduce that the velocities will be inversely as the kinergeties; or in other words, that the momenta (kinergety × velocity) at the end of equal intervals of time will be equal.

Thus, whenever a body is under the influence of a force it possesses potential energy with reference to the other body or bodies concerned in the action. The amount of potential energy possessed by the body will depend directly upon the amount of the force exerted upon it and the distance in the direction of that force through which the body can travel. The same form of statement holds, when, instead of dealing with separate objects, we are concerned merely with parts of

the same object. Thus a body above the surface of the earth possesses potential energy with reference to the surface jointly proportional to its weight W and to the vertical distance H above the surface. Its potential energy with reference to the surface is therefore WH. The earth possesses the same potential energy with reference to the body.

A reference point for motion, force, and energy, frequently employed is the so-called "center of mass" or "center of gravity" of the system. It is not necessary here to discuss the meaning or use of this term in any general way. In the case of two bodies which are made to approach one another by energy which is exerting a force upon each of them, the position of the center of mass remains unchanged by this action. This affords a reason for its use as a reference point.

Suppose the motion of the body and the earth in the foregoing illustration to be so referred. If then the body is set free, the body and the earth mutually approach this point with velocities inversely as their kinergeties, and acquiring energies proportional to the respective spaces traversed. In view of the immense kinergety or mass of the earth as compared with that of any ordinary body on its surface, the velocity and kinetic energy acquired by the earth in the time occupied by the body in its descent to the surface is insignificant as compared with that of the body. Thus in an ordinary case, the motion of the earth is entirely insignificant and the earth may be considered as stationary relatively to the center of mass of the system, and may be used as the point of reference.

If the force is not a uniform one throughout the distance that the bodies traverse in approaching one another, then the potential energy is measured by the total work done upon the body in traversing its path. If we let f denote the mean value of the force while the body is traversing any small distance s in the direction of the force, then the

work done upon the body as it traverses that distance is fs, which is the amount of energy received by the body in traversing that space. The sum of all the successive portions of work fs along the path from the beginning to the end will be the total work done or the total energy received by the body, and will therefore be the potential energy of the body at its starting-point relatively to the final point of the motion. If the force varies progressively, this summation will approach more and more nearly the true value of the work, the smaller we make the successive intervals s. Thus the total potential energy of a body at one point with reference to a second point is measured by the total work which will be done upon the body in passing from the first point to the second point under the force concerned. This work will be negative, that is, will be done *by* the body instead of *upon* it (p. 113), when, in order to move the body from the first point to the second, it is necessary to move it *against* the force considered. In that case, the potential energy at the second position is greater than at the first.

In the case of a force tending to make two bodies approach one another, the potential energy of either relatively to the other is greater the farther apart the bodies are situated, because obviously the amount of work which will be done in bringing either body up to the other will increase with their distance apart. The greatest possible potential energy of one of these bodies with reference to the other will correspond to an infinite distance between the two. Since two bodies cannot be brought without change of form nearer to each other than direct contact, the maximum potential energy of one body with reference to another will be measured by the work done in bringing either body from an infinite distance up to contact with the other body.

Since the force tending to make two bodies approach or recede from one another is in a straight line joining some point of the two, and since the work done upon the moving

body is only in proportion to its displacement in the direction of the force (that is to the component of its motion parallel to the direction of the force), the total work done in moving from one point to another, by whatever indirect path, is the same in amount as by the most direct possible path, namely, in a straight line joining the two bodies. Thus the potential energy does not depend upon the path traversed by the body, but only upon the displacement in the direction of the force.

In the case of a force under which two bodies tend to recede from one another, the considerations are precisely the same as for bodies tending to approach one another, except that the work is done in the opposite direction, and that the maximum potential energy occurs when the bodies are in the nearest possible position to each other instead of in the most remote position.

The foregoing statements have dealt with potential energy in cases where both or all bodies considered were acted upon by the source of energy. But if we consider only one of the bodies, and refer its position and motion to any arbitrarily chosen point, then the body will have potential energy with reference to that point. This will be true, except in the case where the reference point is located upon an object which has always the same motion as the body considered; for, in that case, the potential energy of the body, referred to the point, will be always zero. In the general case, the body, when moving under the force, will have its distance from the reference point either increased or diminished. The total potential energy of the body with reference to the point at any instant will be the work which the body (or rather the energy moving the body) will perform in traversing any path to the reference point from its stated position. This work will be accounted negative, or will be done upon rather than by the body, when the force tends to make the body recede from the reference point.

CHAPTER XI.

MATTER.

Distinction between Matter and Substance. — Aided by the concept of energy, we may examine more closely that of substance. This has been expressed as "that which is assumed as existing in space, and as endued with powers to affect the human senses or portions of itself." But as the powers have been shown to be the forms of energy, we may describe substance as that which is assumed as existing in space, and as endued with its several forms of energy. We may remark first the hypothetical, or at best inferential, character of the concept of substance. Observation shows us that different portions of space at a given time contain different quantities of energy; also that under identical conditions different portions contain and can be made to contain only unequally assorted forms and amounts of energy. Into a given portion can be put, or with it can be associated, a certain definite quantity only of each form. The facts thus briefly alluded to suggest the idea of specific capacities, that is, a definite capacity for each of the forms of energy. But this idea is inconsistent with our notion of space, and the thought therefore arises of something in the space to serve as a receptacle or carrier of the energy, and bearing these capacities as characteristics. Observation further shows that the group of energy-forms may be transferred without change in a continuous path through space. The forms composing such a group appear therefore to be associated more positively than through mere position, as, for instance, by being attached to something occupying that

portion of space. Thus we arrive at the notion of a carrier for energy, existing in space, and possessed of specific capacities for energy of the several forms. The capacities are clearly an intrinsic part of the concept thus arrived at, which is that of substance. The whole concept is clearly supposititious, or if one can find a suitable major premise, it may be called inferential. Its order of certainty, however, is that which attaches to the existence of the human body, or rather to the substance thereof.

Substance, then, whatever be its ultimate nature, must be so organized as to be capable of having energy associated with it in definite ways and amounts. Beyond this point we have pure speculation alone. If more were known of the nature of energy, some further advance might result; but here, too, there is only hypothesis, although there are some indications as to the manner in which some of the forms of energy may be in association with substance. For example, gravitation energy quite possibly acts on the minute portions of substance from without; and likewise cohesion energy. Heat, on the contrary, seems more probably to reside directly on these particles, perhaps as merely the kinetic energy of their random motion. Elastic energy seems to be dependent for its peculiar manifestation upon some structural characteristic of the particles of substance. Of electrical and magnetic energy, perhaps the less said the better; but the former appears external to, rather than as acting within, the particles. Of chemical energy, one may feel some confidence in assigning to structural functions of the particles the selective capacities indicated by quantivalence and combining weights; but as to the seat and nature of the energy which produces and maintains chemical union, and which manifests itself in the production of heat, the mystery is, if possible, deeper. One may hazard the thought, probably not novel, although no mention of it comes to mind, that this energy is identical with that caus-

ing gravitation and cohesion, a thought suggested by, and not obviously inconsistent with, the vortex-atom theory of matter (Part II., Chap. V.).

So far as the foregoing and like considerations can be interpreted, they indicate no very simple character for substance. Form of the particles of substance would naturally first suggest itself as the basis of an explanation of the intrinsic properties of substance. But it is not apparent how mere differences in form could adequately account for the chemical properties, or how form could afford an ultimate explanation of elasticity. Moreover, form in the physical universe is produced and maintained only through the continued action of energy, — usually of cohesion or gravitation. Hence to assume form as the starting-point would not go far towards simplifying our view of substance. No other simple suggestion offers itself. In the entire line of speculations on the nature of substance, there is but one which offers any real resolution of this concept into simpler and well-recognized components. This is the vortex-atom theory of which an outline is given in Part II.

The theory consists in the development of the properties which would be intrinsically possessed by a system of vortex rings in a continuous "perfect fluid," and in comparing them with the known properties of substances. The analogies thus found are most remarkable. The starting-point is a hypothetical perfect fluid, continuous, and uniform throughout space. In the plainest terms, this is merely an imaginary primary substance, intrinsically devoid of all energy, and so simple as to possess no property but kinergety. Minute but multitudinous portions of this fluid are assumed to be in motion as vortex rings. These rings are the analogues of particles (atoms) of substance. In translatory motion, the rings possess kinetic energy because of the kinergety of the fluid, but pass without resistance through it. The vortex ring has elasticity of

form because of the energy of rotation of its constituent fluid. Thus capacity for elastic energy is derived from a mode of motion of this imaginary simple substance; and elastic energy itself would be merely kinetic energy temporarily stored up in the distorted ring. The rings are not only perfectly elastic but are also indestructible, and thus afford all the basis necessary for the ordinary dynamical theory of heat. Being capable of distinct periods of vibration they are adapted to meet the requirements of some radiation phenomena. By their form and elasticity they are peculiarly adapted to the bombardment theory of gravitation and cohesion (possibly also chemical energy?). Finally, through certain structural differences (linkage, beknottedness, etc.) they acquire properties apparently identical with the chemical properties of quantivalence, atomic "weight," and many others. All these points are developed without further assumptions by purely mathematical processes, and they can be disproved only by demonstration of error in those processes, or by contradiction with fact. That further developments of the theory, especially in relation to electricity and heat, are lacking, is due perhaps more to the immense mathematical difficulties attending the process than to inadequacy of the hypothesis. This theory then gives a definite concrete embodiment of the long-standing speculation that the elements might prove to be resolvable into a single, more elementary substance. To attain the degree of simplicity naturally ascribed to it, this ultimate substance must have no property dependent on energy or even on motion. The vortex theory will meet even this severe condition provided that the one property, kinergety, of the hypothetical "fluid" can be shown not to arise from energy. There is no reason for the supposition that it does so arise, but on the other hand no strict disproof is forthcoming. A discussion given in Part II., Chapter V., shows, however, that mere continuous permanent occupancy of

space is an entirely adequate basis for kinergety, at least so far as that property is involved in this theory.

We may then say that such evidence as we have, together with rational and detailed speculative theory, indicates that substance is not simple in character, and gives support to the suggestion of its resolvability into two components. These are energy and an inert underlying substance devoid of all characteristics save kinergety or some even more elementary property to which this may be due.

In view of the avowedly speculative character of our acquaintance with the nature of substance, are we justified in permitting it to exert any influence in the formation of our fundamental physical concepts? As to this, on the one hand, we cannot too strongly insist that no such considerations as those above advanced afford any *proof* whatever of the resolvability of substance. On the other hand, to ignore the possibilities thus opened up would be equally injudicious. To rest the case at this point might be the more logical procedure, but the apparently fixed trend of the mind to seek the simpler under the more complex has established in physics the custom of assigning the name *matter* to the most elementary concept of that which underlies all bodies or substances and out of which they are made up. So long as the doctrine of energy was in abeyance, the concept of matter included not only that of substance as here defined, but also all or nearly all of the "powers." Now, however, matter must be so defined as to exclude the concept of energy. But to accomplish this end as effectually as may be, regard must be had to all reasonable possibilities as to the occurrence of energy as a constituent of substance. It is precisely here that the speculations above given are entitled to place. Not that the denotation of matter should recognize or give expression to the hypotheses underlying any theory, but that the definition should if possible be so framed as to exclude no rational view of

the nature of substance or matter. Should it be remarked that so abstract a concept of matter is a metaphysical rather than a physical one, it may be rejoined that the definitions seek only to reduce the concepts of matter and of substance to the lowest terms, not to analyze them. Such analysis would lie beyond the domain of physics and would fall within the scope of the modern psychology.

Definition of Matter. — If we employ the term *inert* as meaning intrinsically devoid of energy and motion, the following appears to be a definition consistent with the conditions of the problem: —

MATTER IS THE INERT CONSTITUENT OF SUBSTANCE.

There is here no implication that it is physically possible to separate energy from matter. The concepts of energy and matter are mutually exclusive, but this in no way requires or implies or denies the possibility of separation in space and time.

The remark may be again recorded that the concept of matter would become identical with that of substance if the capacities could be demonstrated not to be due to energy. The capacities would then become intrinsic properties of matter, and we should be obliged to recognize some seventy forms or kinds of matter, whereas we now admit these elements only to the rank of kinds of substance. This possibility is not ignored in the above definition, but the presumption seems at present much against such an outcome.

Matter Non-resistant. — It is a common misapprehension that a portion of matter, even when free, resists an effort to set it in motion or to accelerate its motion. This mistake arises from two combined causes, namely, the disregard of the distinction between matter and substance, and the fact that substances or bodies do exert such resistance, or rather that the elastic energy called into play upon them does so.

When a body is pushed by the hand or by another body, we know that it exerts an opposing force due to elasticity, whether or not it be free. The distinction between matter and energy shows that this force can be exerted by energy only and not by the matter of the body. It is true that we do not know any substance devoid of elasticity, that is, one which is incapable of showing some of the phenomena of elasticity under proper test; whence it might be said that we are not justified in assuming that matter can exist devoid of elasticity and therefore incapable of exerting force. The fallacy in this remark lies in the last few words. It is quite possible that matter does not, perhaps can not, exist without elasticity, but none the less it is the energy and not the matter which exerts the force. Even granting what has not been proved, that matter does not exist without elasticity, the independence of the two concepts, matter and energy, is in no way impaired thereby, and there is no reason for confusing them hopelessly, as would be done by attributing to matter the exclusive attribute of energy, — the exertion of force.

When force is exerted against a body, the latter does not acquire its full velocity instantly, but its speed increases only at a definite rate. This is sometimes cited as indicating a "resistance" of matter to being moved, and also of the "inertia" of matter. It indicates neither. By our definition of force, we recognize the fact that when a free body moves under a force, it acquires kinetic energy at a definite space-rate, no more no less, at every point of its path, this space-rate being used as the measure of the force. A freely moving body, therefore, can have its motion accelerated under a certain force at no greater rate than will correspond to this space-rate of transference of energy to it. The body will be accelerated at such a rate that, supposing it to start from rest, its velocity at any point of the path will be such that the kinetic energy of the body will be

equal to the energy which has been imparted to it, namely, the force multiplied by the distance through which the body has traversed in the direction of the force. Since the kinetic energy is proportional to the kinergety and to the square of the velocity of the body, this velocity at any given point of its path, or the time of acquiring any specified velocity, will depend upon the kinergety of the body. A body with greater kinergety, after having traversed under the same force a given distance along its path, will possess the same energy and a less velocity than a body of smaller kinergety.

If the matter of the body exerted an opposing force, that is a resistance, kinetic energy would be acquired at a rate proportional only to the difference between the two forces, which is contrary to the fact. At first glance this last statement may seem counter to the admission that an actual body even if free does exert a resistance equal and opposite to that exerted against it, as for instance in the collision of elastic bodies. But not only is this resistance, as just shown, due to the elastic energy called into action upon the body being moved, but also this energy is only the transmitting agent between the acting source and the matter of the body; that is, the elastic energy exerts an unresisted force on the matter of the body and in the direction of the external force upon the body. To associate with the concept of matter any idea of resistance to motion is not merely flatly contradictory to the entire line of presentation adopted in this book, but is irreconcilable with any consistent view of the phenomena of physics.

Properties of Matter. — The many properties of substances need not be here catalogued in full. To mention extension, inertia, elasticity, cohesion, gravitation, magnetization, chemical affinity, electrical conductivity, etc., is sufficient to indicate what is meant. If the foregoing distinction between matter and substance be disregarded, intentionally or not, the terms become coextensive. This custom, however

clearly reprehensible, has been and still continues to be almost universal, as also has the corresponding omission to discriminate between properties of matter and of substances. But as the ideas of matter and energy are mutually exclusive, any property demonstrably due to the presence of energy is therefore not a property of matter, but only of substance. Further, no property of substances can be rigidly ascribed to matter, unless demonstrated not to arise from energy. Bearing in mind the inferential character of the properties which we have called capacities, our small knowledge of their nature if they do exist, and also the rational analogies to them based on the mere motion of a hypothetical inert "matter" without energy except in virtue of its motion, we must deny place as properties of matter to all the properties of substances, at least pending proof, except possibly to extension, inertia, and kinergety. These must be further considered.

Extension. — As to extension, the volume of bodies is determined in part at least by the quantity of heat energy which they contain. What the volume of any body devoid of all heat energy would be, is a matter of speculation; and whether such volume would be the volume of the matter contained in the body, or even whether it would indicate that this matter when devoid of energy, possessed volume, cannot be affirmed. The property of extension, that is, the occupation of space, is not deducible as a property of matter from the fact that substances occupy space. It is in reality only through speculative considerations of a sort to be alluded to in Part II., that this property has been ascribed to matter. We may, therefore, properly deny that it possesses a legitimate claim to such recognition.

Inertia. — The so-called property of inertia is in reality not a property but a principle or doctrine (p. 19). Viewed under the modern definitions of matter and energy, the principle falls to a position of comparative insignificance. Any-

thing which changes the state of motion of a body is, by definition, energy. Matter cannot, therefore, produce such a change in state of motion, and hence is inert. Thus the principle is a direct outcome of the definitions. It must however be noted that the concept of capacity for kinetic energy (kinergety) is often associated with or merged into that of inertia. Bodies are said to possess their kinetic energy in virtue of their inertia, and to "resist motion" also because of that inertia. This practice results from a confusion of ideas. Inertia and kinergety have no relation whatever to one another.

Kinergety. — The property which we have called kinergety appears to belong universally to all portions of substance however minute, and to be inseparable from them. It appears in no way to depend on any amount or form of energy in the substance, but of course we can only assert this proposition as true within those limits inside of which we are able to remove or add to these quantities of energy. If it should be true, as some theories suggest, that substance can exist as such only through the association of some form of energy with some portion of matter, or through some mode of motion of matter, then it is conceivable that the kinergety of the substance might result from some other property in the underlying matter, — such for instance as mere permanent occupation of space. However much or little weight we may assign to such hypotheses, as explanations of the real nature of matter and substance, still, in view of the possibility which they present, we must decline to admit that the possession of kinergety by matter is a *necessary* inference from the possession of this property by substances. It may be well in view of the future use which is to be made of it, to restate here some of the evidence upon which kinergety is classed as a universal attribute of substance.

A body, moving with a definite velocity with reference to a given point, possesses a definite amount of kinetic energy

with reference to that point (p. 60). But at the same instant it possesses with reference to any other point a definite other quantity of kinetic energy, the difference being determined solely by the difference in velocity of motion of the body relative to the two points. The kinergety of the body is of course in no way affected by the mere change of reference point of the motion. Again, a body undergoing no apparent change, *e.g.* a piece of metal, possesses at different times always the same kinetic energy when its velocity is the same. Again, if we add to or abstract from such a body as the piece of metal just cited any amount of substance, we find that at the same velocity its kinetic energy is respectively increased or diminished (within the limit of sensitiveness of the methods employed). The converse of this statement is also true. Again, the kinetic energy of a body which undergoes no perceptible increase or diminution of substance is the same at the same velocity, whatever the external condition, and whatever the amounts of energy of other forms which it may possess. These propositions are established upon no simple experiments but are confirmed by their consistence with universal experience in all branches of physics. Beyond this we have the general principle of the conservation of kinergety (p. 156). From all these, the inference is unmistakable that kinergety is an intrinsic property of all substances and of every minutest portion of them. It appears also that the kinergety of any portion of substance is a property wholly independent of the existence of any other substance or body, and of any other property or form of energy.

Gravitation. — It may be well to consider somewhat further such a property as the capacity for gravitation energy. The action of gravitation is clearly due to a form of energy. The capacity for gravitation energy may be defined as being the energy which a given body would acquire in moving through a specified distance in the direction of a second

body, the two being at a specified distance from one another, and no other bodies being concerned in the action. If the second body and the distances were to remain the same, and the first body were replaced successively by others, the quantity of energy received by each of these several bodies would be in general different for each of the bodies, and would be the same for any one of them at all times. These quantities may be taken as the relative capacities for gravitation energy of the respective bodies. But it must, of course, be understood that these quantities are a function not only of these bodies but of the second body which remains constant in all of the supposed cases. These capacities we know by Newton's law for universal gravitation to be directly proportional to the kinergety of the respective bodies. For although Newton's law as ordinarily expressed states them to be proportional to the respective masses, the term *mass* as there used really represents what we have called kinergety or capacity for kinetic energy. The capacity may be otherwise defined, as for example, by stating that the capacity of a system of two bodies is the amount of energy acquired by them in moving through a specified distance towards one another when at a specified distance apart. As so defined the capacity is by Newton's law proportional to the product of the kinergety of the respective bodies. Or the capacity may be treated from other points of view but with substantially the same result. This capacity for gravitation energy is as intimately associated with every minutest portion of substance as is kinergety, and so far as known is also universally so associated, but there is even better reason for refraining from the inference that it is a property of matter than in the case of kinergety. The plausibility of the hypothesis which explains gravitation as due to the kinetic energy of a certain system of moving particles of substance, combined with the argument just presented in the case of kinergety to the

effect that a property may attach to a portion of substance, and still not attach to the inert matter composing that portion, indicates clearly that matter itself may possess no capacity for gravitation.

Mass: Quantity of Matter. — Whatever concept of matter be adopted, the idea of quantity is associated with it. This idea is denoted by the term *mass*, — a term which carries with it no further significance whatever. Hence,

MASS IS QUANTITY OF MATTER.

It is not a property of matter, neither does it refer to any such property. Although often regarded as arising from or related to inertia, it has absolutely no connection with that principle. The term is nothing more than a verbal substitute for the words *quantity of matter.*

Inasmuch as we are and can be acquainted with matter only as manifested in substance, quantity of matter can be measured only through some property of substance. In other words we can measure only quantity of substance. To give recognition to this fact by defining mass as quantity of substance rather than of matter would be more logical. But custom has fixed the latter usage, through the long employment of matter, with the denotation more strictly pertaining to substance, as above shown. This is perhaps not a serious disadvantage; for if substance differs from matter, it does so through being matter organized through motion or energy which is in permanent association with it. In this substance thus organized into the possession of properties, it is often assumed that it is only with the inert component that we are generally concerned when measurements of quantity are to be made, that is with matter. If substance bears to matter the relation supposed, then a designated quantity of substance consists of a definite quantity of matter in permanent association with a definite quantity of energy or motion. Thus in whatever manner

quantity of substance be measured it will bear some definite relation to quantity of matter. The nature of this relation will, however, depend on the method used. Hence the selection would naturally be made with a view to securing the simplest possible relation, namely, that of direct proportionality. If we had been able to demonstrate with certainty any one property of matter, this would without question be made the basis of the definition of quantity of matter, that is of mass, even if not adapted to the practical measurement of mass. In default of this, a property of substance must be chosen. The conditions to guide the choice are: First, the property should be such as to render highly probable the assumption that its adoption will yield quantitative measurements directly proportional to mass; second, the property must be such as to lead directly or indirectly to a practical method of measurement of demonstrably sufficient accuracy. Let us now proceed to the selection.

The several "powers" of substances, even if regarded as properties of substance, are debarred from service, as the amount of any of them associated with a given portion of substance varies with time and place. The choice is therefore restricted to the "capacities." Of these the chemical properties are obviously unsuitable, as they are not common in either kind or degree to all substances, and can obviously bear no common or simple relation to mass. The same is clearly true of the electrical, magnetic, elastic, cohesive, and heat properties, as well as of properties related to radiation. The choice, thus narrowed, lies between kinergety and capacity for gravitation energy, or some manifestation of it (such as weight).

If a homogeneous body, that is, one of which every portion is identical in properties with every other portion (so far as can be discovered), be cut into parts of equal volume, these parts would naturally be assumed to contain equal

quantities of matter, that is equal masses. This appears to be the most primary view attainable of equal masses. But by weighing (p. 104) these equal volumes will be found to have equal weights and therefore (p. 109) equal kinergeties. Hence there is strong presumption that equal masses of the same substance have equal weights and equal kinergeties. No such direct evidence exists as to the relation of the weight or kinergety of a given mass of a substance of one kind and that of an equal mass of another, for the obvious reason that equal volumes of different substances probably have masses which are not only unequal but stand in no primarily determinable relation to each other. We have therefore to resort to more indirect considerations, and these are furnished by a review of the properties of kinergety and gravitation. This, however, has already been made (p. 144). We have seen that kinergety pertains to every portion of substance; that there is nothing to indicate that it may not be an intrinsic property of matter as well as of substance, or at least the direct result of an intrinsic property of matter, since it does not appear to require either energy or motion for its explanation; that as a concept it is essentially simpler than that of capacity for gravitation energy; that it is capable of primary measurement with moderate accuracy, but of indirect measurement with great accuracy by "weighing" (p. 104). As to capacity for gravitation energy, we have found it less simple in character although no less universal as an attribute of substance. But all the facts concerning gravitation would be entirely met by the supposition that this capacity does not attach to matter but only to substance. Neither is the capacity itself, as above defined, any more directly measurable than kinergety. Weight as a manifestation of gravitation might be suggested as a substitute for the capacity for gravitation energy as a measure of mass. This is based on the common assertion that weight is pro-

portional to mass. But for the reason just stated in considering the capacity itself, this is evidently pure assumption. Moreover weight varies with locality, and therefore cannot be proportional to mass, which is constant.

There is thus a distinct preponderance of reasons for the selection of kinergety as the property through which to define quantity of substance and upon which to establish the measurement of mass. This choice has also the sanction of custom. *We therefore assume that mass is directly proportional to kinergety.* Also as weight has been demonstrated to be directly proportional to kinergety, *we assume that mass is directly proportional to weight* — the weight being always understood as taken at or corrected to a fixed point of the earth's surface. The propositions just enunciated may be stated in another form as follows: *The relative masses of bodies are assumed to be to each other in the ratios of the kinergeties of the bodies, or of their weights at a given locality.* This statement brings out the point which is to be clearly appreciated, that like all other physical quantities, mass is purely relative. Masses are referred to a chosen standard of mass, the customary standards having already been named.

The existence of a broad assumption is generally overlooked in asserting the proportionality between mass, kinergety, and weight. The assumption exists whatever form we give to the concept of matter, and is not a result of this particular mode of presentation. The oversight arises from failure to recognize kinergety sufficiently or at all as a distinct property of substance. The kinetic energy of a body is ordinarily said to be proportional to its mass, with no allusion to kinergety. Hence inevitably arises the mistaken idea that mass is a *property* of substance or matter. There is perhaps no more prevalent misconception in the range of physics.

It is to be noted that we arrive at the same results

whether relative weights or relative kinergeties be adopted for the measurement of mass, because of the proved proportionality between weight at a fixed point and kinergety. But it is also to be borne in mind that weight is merely a *force* whose intensity (cf. p. 101) varies with locality, so that bodies of equal mass or kinergety do not have equal weights at different places on the earth. Equal weights at different geographical localities do not therefore measure off equal quantities of matter or of substance.

As presumptive evidence of the correctness of the assumption of direct proportionality between mass and kinergety, may be cited the comparative simplicity of the laws of matter and energy framed on that basis. Complexity might naturally be assumed as the sure result of an error in this fundamental assumption, although such a supposition is not capable of proof.

Practical Measurement of Mass. — All physical phenomena arise from the action of energy associated with substance. Hence most physical and chemical measurements have to do with quantities of energy associated with quantities of substance or matter. A sufficiently delicate practical means of measuring quantity of substance or quantity of matter is thus a necessity. The usefulness of such a process is enhanced by the fact that under given conditions of temperature, etc., a stated quantity of a given substance bears always the same quantities and forms of energy; so that the measurement determines incidentally a precise quantity of energy. The practical measurement is a process which enters so fundamentally into every kind of quantitative work that higher delicacy is demanded in it than in any other. Fortunately this is attainable without undue labor by the use of the equal-arm balance (p. 104) in the process called "weighing." The limit of accuracy is from one part in one million to one part in ten million (p. 105). The process has already been sufficiently described.

Weightal *vs.* **Weight** *vs.* **Mass.** — As the process of "weighing" makes use of the force called weight, and consists in offsetting the weight of the given substance against that of standard bodies, the resulting quantities of substance are often called "weights" of the substance rather than masses. This usage is sometimes insisted upon as the only one consistent with the nature of the process as just sketched. But except in the rarer cases where the object is to ascertain the amount of the *forces* involved in its action, this usage seems misleading and mistaken. It is true that weight is employed in the operation, but only as means to an end, just as is the beam of the balance or the elastic force called forth in the beam. It is true that the portions of substance measured out have equal weights at the same locality, but we are in general in no wise concerned with that fact except as a convenient incident. What we are seeking is to ascertain the quantity of substance with which we are dealing, or to reproduce a stated quantity. This quantity would be the same in any part of the universe, while its weight would vary with its locality even to the extent of vanishing altogether. Even on the earth's surface, the weight of a standard body differs to the amount of one-half of one per cent, an amount quite inadmissible as an error in ordinary scientific work. But if the chemist, for instance, were really to measure off equal weights of a substance (as would be done if the spring balance were used), he would obtain quantities of substance differing to some such extent as this according to locality. In work involving only percentage composition, all weighings being made at the same place, no resultant error would be thus introduced; but in almost all other chemical and physical measurements it is essential that the results of "weighings" should be comparable at all places and times. Hence it is not with weight that we are concerned. The customary instrument, the equal-arm balance, employed in the usual manner, yields directly what

we want, and its results are comparable wherever the observations are made. The quantities resulting from its use are now commonly called "masses" by physicists. The terms *atomic masses, combining masses*, etc., are likewise used instead of "atomic weights," etc.

But notwithstanding the prevalence of the use of the term *mass* among physicists in recent years, conservative opinion, notably among eminent chemists, is opposed to the employment of that term in place of the time-honored "weight" to denote quantity of substance or matter as measured by the process of weighing. Thus, not only is "weight" still universal colloquially, but it is common with chemists, and is even used by physicists when referring to quantity of substance without special reference to the property of kinergety, with which mass is often confounded; and further, "weight" is still employed by preference by many eminent authorities, as just indicated; the terms *weight, atomic weight, combining weight*, etc., being insisted on instead of *mass, atomic mass*, etc. As the consistent and judicious use of terms is of no less scientific than pedagogic importance, we will review the considerations which may influence the choice in the present instance.

The trend of the foregoing paragraphs on quantity of matter and the properties of matter shows clearly the supposititious character of the assertions that mass is proportional directly to either weight or kinergety. To employ *mass*, therefore, to denote quantity of substance as measured through weighing, is to introduce into terminology an inference which can be demonstrated by neither logic nor experiment. This appears to be the sufficient ground for the objection above cited. We may thus justly protest against the assertion of page 147, that "it is only with the inert component [of substance] that we are generally concerned when measurements of quantity are to be made, that is with matter."

The employment of *mass* when referring to quantity of substance with regard to its kinergety is equally objectionable.

On the other hand, to designate quantity of substance as a "weight" when referring to either the quantity merely or to its kinergety, is equally open to criticism. For the term *weight* denotes a force, not a quantity of substance or of matter, and it should be restricted to that denotation. It is the plain duty of science to prevent the perpetuation of the embarrassment now so great to students and members of the technical professions through the prevalent use of "weight." But it is equally clear that *mass* is not a reasonably acceptable substitute. With full willingness to submit them to the censorship of scientific criticism, the terms *weightal* and *kinergety* have here been proposed,—the former to replace the terms *mass* and *weight* in denoting quantity of substance as measured by the equal-arm balance by weighing against the usual standard "set of weights"; the latter to be employed instead of *mass* when quantity of substance is referred to with respect to capacity for kinetic energy. The service which the term *kinergety* can render has, it is hoped, been in some degree made evident through its employment in the pages of this book. As to *weightal*, some further remarks may be added.

Operations and laws in all branches of the natural sciences are based upon the measurement of quantity of substance by the equal-arm balance or by processes equivalent to this in result. To designate quantities thus measured may it not be a safe and practicable middle course to use the term *weightal*?[1] By derivation, the word is, of course, properly an adjective, a weightal quantity being one taken with respect to weight. But as in the case of "residual" and other like terms, we may properly use the adjective as a substantive. Restricting by definition then the term, as we may, to

[1] A term to which the purist may object.

mean only quantities taken with respect to weight measured by the equal-arm balance: —

WEIGHTAL IS QUANTITY OF SUBSTANCE MEASURED BY THE EQUAL-ARM BALANCE.

The term thus will denote precisely what the physicist and chemist wish to indicate when they refer to quantities of substance now commonly denoted sometimes as "weights," sometimes as "masses." It will, however, denote neither more nor less than is desired, and will introduce no assumption, while at the same time serving to remove confusion. The verbal transition from *weight* to *weightal* is so slight as to render the change an eminently practicable one; while the distinction is abundantly sufficient and obvious, whether in the printed or spoken word. Perhaps *atomic weightal* may find acceptance where *atomic weight* and *atomic mass* receive just opposition.

Molecules and Atoms. — In following out any hypothesis as to the ultimate constitution of bodies, it is found necessary to treat very small parts of the body or substance as individual particles. All hypotheses possessing much weight at the present time regard these small portions or particles as distinct aggregations of matter more or less permanent in character; and the molecular hypothesis assumes them in general to be separated from each other by distances usually many times the diameter of the particles. In this molecular hypothesis, these particles are called *molecules;* and the single *molecule* is regarded as the smallest portion of a substance, whether chemically simple or compound, which can exist without loss of the characteristic properties of the substance. The molecule, in its turn, is in general made up of two or more atoms, these being of the same or different kinds, according as the substance is simple or compound. The *atom,* in its turn, is the smallest portion of an elementary substance which can exist. It is

not held that the atom possesses, in the same degree as the molecule, all of the properties of the substance.

It is not uncommon, indeed it is rather customary, to speak of these molecules as molecules of matter. This is admissible in the sense that they are composed of matter; but as viewed from the attributes ascribed to them, namely, the possession of various properties, the molecules may possess, or have associated with them, energy of various forms. For instance, the possession of the property of elasticity is ascribed to all molecules and atoms. Until we are able to account for this elasticity in some way which proves it to be merely a combination of motion with matter, we shall still be dealing with matter associated with something of whose nature we are utterly ignorant. The vortex-atom theory affords the nearest approach yet made to such an explanation.

Conservation of Matter. — The following generalization is derived from a vast range of experience: —

The kinergety of every portion of substance is constant and permanent.

If we assume, as is customary, that the kinergety is proportional to quantity of matter, that is, to mass, then this proposition may be stated in the form : —

The mass of every portion of substance is constant and permanent, i.e. *is conserved.*

These statements may be otherwise phrased, as, for instance, by saying that the total mass enclosed within any fixed boundary through which no substance can enter or leave during the time under consideration is constant; or again, substance or matter can neither be created nor destroyed. This is one of the most important and far-reaching generalizations of physics. The sort of evidence upon which it rests may be in part indicated as follows: —

Weightal is proportional to kinergety. Kinergety is constant. If a body be separated by any means, physical or

chemical, into any number of portions, the total weightal of those portions will be the same as the weightal of the whole before the separation. Therefore the total kinergety, and hence the total mass, of all the parts is the same as that of the whole body before the separation. If two or more substances are made to combine chemically, the weightal, and therefore the kinergety and the mass, of the resulting substance or substances is the sum of the weightals, kinergeties, or masses of all entering into the action. Whatever change of properties a body or bodies may undergo, the total weightal of the involved substances at the end of the operation is equal to the total weightal at the outset. This line of proof extends only to such substances as we are able to weigh. The phenomena which involve transference of kinetic energy, or any in which variation of kinetic energy of a given body would cause perceptible change in the phenomena, are found to be consistent with the general proposition of conservation. The constancy of the periodic times and of the orbits of the planets affords further evidence of the constancy of kinergety and mass in those bodies and in the sun.

The principle of conservation is generally held to apply not only to matter, but to each element or elementary substance in particular. While we cannot claim this view as demonstrated, still it is generally held, or at least the view that the elements retain their individuality in some degree. If we adopt this point of view, the doctrine of conservation extends to the elements. If we reject it, we must restrict the statement of the doctrine correspondingly, that is, so that it shall apply merely to matter.

CHAPTER XII.

COMMENTS ON CERTAIN DEFINITIONS.

Remarks. — One who suggests departures from established custom in scientific definitions, must furnish reasonable demonstration of the validity and the necessity for the proposed changes. Complete proof of the validity of the definitions offered in the foregoing pages is possible only by applying them to all related physical knowledge, — a task impracticable within the compass of a brief and plain exposition such as is here undertaken. To the outline of proof afforded by the preceding chapters, it may here be added that such further criticism as the author has been able to bring to bear has not detected any want of sufficiency in the definitions. The necessity for change will be clear to any one who will observe the incoherent and inconsequential character of the definitions current in even the best text-books. Scrutiny of about forty of these has brought conviction on that point. Whatever of clearness and logical sequence the suggested changes have rendered possible in the presentation of the subject-matter of this book, is the best kind of evidence of the need and correctness of those changes.

It may not be amiss to cite here as a cardinal principle of pedagogics, as well as of science, that no term should be used in science without a clearly assigned denotation, that is, without having been clearly defined. However broad and vague is the connotation of the term, its denotation (p. 7) must be perfectly sharp. This is always possible, al-

though often neglected and sometimes denied, — notably in the definition of "matter." It is true that we know little or nothing of the nature of matter, but we are not therefore unable to give to the term a perfectly precise definition. If we were, the term should have no place in science. We employ it to stand in place of a definite idea which we can state (cf. p. 10), and that statement constitutes the desired definition. The sundry attributes form no part of the definition or denotation. The more the nature of matter becomes known, the better can its definition be made, of course, but this is no excuse for present lack of clearness. Accuracy in the definition and employment of terms is a prime duty imposed by considerations of economy of effort.

On Definitions of Matter. — To define matter as "that out of which bodies are made up" fails in two chief respects. First, this denotation includes energy as well as matter; for every known body contains as essential components many forms of energy. No theory of matter even exists which does not ascribe to the ultimate particles of substance some one or more forms of energy, or forces which must be due to energy. In some of the theories, the force or energy is boldly engrafted in the desired form upon the inert matter, in others, a special mode of motion is imposed upon the matter by which it is aggregated into separate, discrete particles of substance. In all cases, the energy is there. Second, this class of definitions fails to indicate the inferential character of the idea denoted by matter.

Perhaps the most strongly intrenched definition is that given by the phrase "that which occupies space." *Speculations* on the nature of matter strongly support the view that this may ultimately prove to be the correct or at least a partial definition. But for the present, one may perhaps be permitted to question its availability. Bodies occupy space. But with a degree of certainty at least equal to our

knowledge of their existence, we know that this extension is due in large part to the energy which keeps their particles at a distance from each other. There is indeed no direct evidence that matter devoid of energy does occupy space. The view that it does so is purely a hypothetical deduction. This fact alone is enough to debar this definition from use in elementary physics. A definition based upon a remote, specific hypothesis not presentable to a beginner is certainly not adapted for use in elementary physics.

Some modern text-books follow Maxwell's lead of thirty years ago, and define, or rather describe, matter as the "vehicle of energy." With a qualification to indicate the inferential character of the idea of energy, this may serve fairly well, although lacking something in definiteness. But it presupposes an adequate definition of energy, which is rarely if ever provided.

On Definitions of Energy.— The current definition of energy is usually given in the words, — Energy is the capacity of doing work. Therefore in any orderly presentation of the subject, a definition of work must precede that of energy. But the accepted definitions of work are either of the form, — Work is the action of moving a body against or by a force; or of the form, — Work is any process of transference or transformation of energy. These have been fully and formally given in the chapter on Work. Now force can be adequately defined only in terms of energy, as at page 41, and as will be further shown in the next section. Thus the definition of energy in terms of work leads merely to the vicious circle, — energy, work, energy. It must not, however, be supposed that any such deficiency in logic is openly spread upon the pages of the text-books. The definitions and explanations of force there given furnish the student with so inadequate a view of its nature that he is generally quite unaware of its real relation to energy, and thus fails to recognize the fallacy.

The claim might be urged that it would be possible to define work in such terms as to remove this difficulty; and that to do so would be desirable, since it would reënforce the above long-established definition of energy. However advantageous this might be, it appears not to have been undertaken, and its success is at least doubtful. For instance, work might be said to be the operation of changing the state of motion of a body or of maintaining its motion when not free. This covers all cases of work, and in so far would suffice. But some difficulty might be found in suitably defining "free," and apart from this the suggested form is easily seen to be far inferior to the existing ones in adaptation to quantitative measurement.

The formulation of energy as the power to change the state of motion of a body, is free from the logical and other embarrassments attaching to the current definition, and to the suggested change in the definition of work. It expresses the concept of energy in terms so simple as to be intelligible even to persons unfamiliar with physics, — a point of some importance in the extension of the correct colloquial use of the word. Not less important is this simplicity in the scientific presentation of the subject, as it permits the introduction of the idea of energy in advance of that of force, or work, or even of matter. That this early introduction is not only logical but of great help in the elementary presentation of physical science, seems not to be duly recognized. It is hoped that the latter point may have received some illustration in the first part of this book. That the leading position in sequence as well as in importance logically belongs to energy, may be briefly indicated as follows. Everything which we observe, that is, all phenomena and observed properties, are due to energy or its changes of form or location. All experience has therefore to do with energy, and with that alone. As it is foremost and ubiquitous in experience, so should it also be, if possible, in the

treatment of physics. Two or three decades ago, this was impossible; but it is so no longer. Force and work must now be dealt with as actions of energy, and matter must be regarded, for the present at least, as an inference one step more remote than energy.

The fact that the majority of phenomena are not recognizable as due to "change in state of motion of bodies," might suggest itself as indicative of inadequacy in that form of definition. But although we do not know that such phenomena *are* due to change of motion, we are equally unable to show that they *are not* so. Moreover, it is not at all essential that the denotation of a term should cover all the attributes of the term, that being the function of the connotation. The criticism would therefore not be justified. The adequacy of the definition can be called in question only by its failure to include as energy anything otherwise correctly demonstrated to be energy; by failure to exclude anything known otherwise not to be energy; by leading to inferences contradictory to known facts; or by failure to lead up to a rational primary method of measuring quantity of energy. Sufficient evidence on these points has been given. The necessity noted in the last of them appears to be the cause of a tendency in some careful writers to advocate definitions which are really denotative not of energy but of *quantity of energy*. A similar tendency is much more pronounced in the case of force. Such a practice, however harmless in mathematical physics, is inadmissible in a fundamental treatment.

To define energy as that which is (or through its changes is) the cause of all physical phenomena, is unsatisfactory. The idea conveyed is too vague and intangible to be of service to the uninitiated, and fails to lead directly to any primary method of energy measurement. Its definition as, — That which underlies physical phenomena and which is constant in quantity throughout its many changes in form, is simi-

larly unsuitable. Both of these are descriptions rather than definitions.

On Definitions of Force. — Two centuries ago, Newton employed the term *vis impressa* with entire clearness to denote precisely the kind of quantity now generally termed *force.* Newton's definition of the term is here quoted from the "Principia": —

Def. IV. — "Vis impressa est actio in corpus exercita, ad mutandum ejus statum vel quiescendi vel movendi uniformiter in directum."

Somewhat literally rendered this is: —

Def. IV. — Impressed force is action exerted upon a body to change its state either of resting or of moving uniformly in a straight line.

The term *vis impressa* and its translation *force* and *impressed force* are now given up by universal consent to the denotation rendered classic by the "Principia," but until a few decades ago the three terms, *vis, force,* and *Kraft,* now properly held to be equivalent, were loaded with the added significance now denoted by *energy,* — at least so far as the latter concept was then apprehended (cf. p. 76). In the somewhat sentimental revulsion of feeling which has attended the effort to establish the proper discrimination between the denotation of force and energy, there has been manifested a disposition to deny to force any adequate recognition. Before dealing with this point, however, let us examine Newton's definition in detail.

The words *statum vel quiescendi vel movendi uniformiter in directum* are the exact equivalent for the phrase "state of motion" as defined at page 15. Newton's definition may then be rendered by: —

Force is that action exerted on bodies to change their state of motion.

In the employment of *actio,* Newton must have intended a clear distinction between action and agent. Also he ap-

pears to have used *actio* in the sense in which *action* has been employed in the definition of force in this book (p. 41); namely, as denoting an operation. It is therefore pertinent to inquire, — "action" of what? It seems altogether improbable that Newton possessed an adequate answer to this question, or that such an answer could have been given by any one before the middle of the present century. About all that modern science has contributed to the extension of the concept of force lies in that answer, and yet this contribution, great and necessary as it is, is but indistinctly suggested in even recent text-books of physics. Action of what? Action of energy. Insert then the words *of energy* in the Newtonian definition, and it then covers the modern concept of force. But it also covers more than that concept; it includes also the concept of work. Newton's phraseology "to change its state," etc., includes both the production of a *tendency* and of the *change* also. Now the latter action is work. Hence the necessity for the introduction of the word *tendency*, in the proposed definition. If the foregoing comment is substantially correct, the employment of the Newtonian definition at the present time without modification is to ignore the entire progress of two centuries in this direction.

A distortion of Newton's definition, to which attention should be directed, consists in the substitution of "that which" for action. Thus modified, the words form a definition of energy, not of force, and have been far from helpful to the student.

Force has also been defined as "tendency to acceleration." But this formula, like the others already referred to, fails to indicate the known relation of force to energy. Also it confounds the action, which is force, with the result, which is not force but is the effect produced by the acting energy through its action, force. The employment of "acceleration" instead of "change in state of motion" is admissible,

on the ground that all change in state of motion appears as acceleration when referred to a proper point (cf. p. 15). But the brevity gained by the substitution is more than offset by the loss of directness in application to phenomena.

For many years, two other force definitions which are equivalent to one another have been current, although by no means universally acknowledged to be adequate. They state force to be, — "the space-rate of transference of energy" and "the time-rate of change of momentum of a body." The fundamental defect in both is that they define, not force, but *quantity of* force. The practice of employing definitions of quantity in place of those of the abstract concept has found many followers, but one may be permitted to doubt whether the principle involved has been duly recognized, or indeed even the bare fact itself. The definition of anything which is capable in the concrete of quantitative measurement should, to be sure, lead up directly to a consistent primary method for that measurement; but this proposition in no way justifies the principle of substitution above stated, nor does there seem to be any justification therefor. Moreover, the space-rate and time-rate definitions of force fail in two other ways. First, they assign to force no recognition as a physical action, when motion is not in progress. In other words, they ignore the existence or occurrence of force in static phenomena. Of course the text-books which thus deny recognition to balanced forces in the definition do not consistently omit the discussion of them in the phenomena in which they occur. Second, these definitions are further lacking in that they ignore, and by implication deny, that there is any interpretation to be given to force other than a purely mathematical one. Such a position is not consonant with the generally adopted attitude of mind towards physical phenomena, energy, and matter. The concepts, respectively, of a thing, of that thing in action, of an action by that thing, and of an effect produced by

the thing through the action are perfectly familiar and rational. The relation between these three is precisely the relation between energy, force, and the effect of the energy through this action, whether the effect be change of motion or merely tendency to that change. It is just this relation which the mode of presentation adopted in this book will, it is hoped, render more clear; as from this standpoint the concept of force becomes duly subordinated to that of energy, and falls naturally into its useful subsidiary place.

Part II.

SPECULATIONS AND THEORIES ON MATTER AND ENERGY.

"NATURAL SCIENCE considers the world a mechanism, and for that purpose transforms the reality in a most complicated and ingenious way. It puts in the place of the perceivable objects unperceivable atoms which are merely products of mathematical construction quite unlike every known thing; and nevertheless these atoms are scientifically true, as their construction is necessary for that special logical purpose. To affirm that they are true, means that they are of objective value for thought. But it is absurd to think, with the materialistic philosopher, that these atoms form a reality which is more real than the known things, or even the only reality, excluding the right of all not space-filling realities. . . . There is, indeed, no physical object in the world which natural science ought not to transmute into atoms; but no atom in the world has reality; and these two statements do not contradict each other.

". . . [It is not wiser than] to cast up against the physicist that his moving atoms do not represent the physical world because they have no color and sound and smell. If they sounded and smelled still, the physicist would not have fulfilled his purpose." — HUGO MÜNSTERBERG, in *The Atlantic Monthly*, May, 1898.

CHAPTER I.

FUNCTION OF THEORY AND HYPOTHESIS.

Value of Theory.—In view of the diversity of the elements and of the forms of energy, it is entirely within the province of physics to inquire in what this diversity consists, and what is the nature of matter and of energy, the answer to the latter involving the explanation of force.

Hypotheses, even imperfect and insufficient, are not only helpful, but in fact are the only stepping-stones by which we can advance in this direction. Their function is to point out a possible relation between a multitude of observed phenomena, which shall serve to unite these into a connected system. Such a system, although it may be wide of the truth and may prove ultimately to be entirely incorrect, may yet be of the utmost service for the time being. The more general view of the phenomena thus obtained enables the mind better to grasp them in their relation to each other, and suggests lines of investigation which, if carried out, contribute just so much to scientific knowledge. These new studies will accumulate added facts which will either confirm and extend or contradict the hypotheses which led to their discovery. In either case, progress has been made, and the usefulness of a hypothesis may be as well demonstrated by its overthrow as by its confirmation. Value of theory to the student of physics, through its cultivation of the scientific use of the imagination, is of the highest importance; and the continual development of novel suggestions and possibilities reduces the danger of narrow or biased interpretation of fact. The

remarkable stimulus to the development of new branches of mathematical inquiry, through their application to physical theories, bears further testimony to the usefulness of such theories. As an example of these three points, one has but to consider the vortex-atom theory, and to contrast one's own thoughts upon the nature of matter before and after a careful study of this theory. But in order that hypotheses, and theories which are developed from hypotheses, should possess their highest efficiency, it is requisite that they should be regarded from an entirely judicial point of view. No theory can ever deserve that order of credence which attaches to an observed fact. All theory is based on logical inference, in the process of which are always involved some propositions or hypotheses which are incapable of absolute demonstration. All theories must, therefore, be regarded as at best tentative; and when the narrow range of human knowledge is recognized, they certainly must be held to be entitled to a very small degree of real confidence; they certainly can be no more than the crudest approximation to the ultimate truth. If this estimate be placed upon their value and they be employed merely as temporary structures to be replaced by better at the earliest possible moment, they may serve a most helpful purpose. This is certainly the only scientific view to be taken of them, and those persons who found upon the existing knowledge of the physical universe, — a knowledge in which theory plays so large a part, — a superstructure of materialism, must certainly be considered as utterly unscientific. To deduce materialism from physical science demands either an illogical mind or procedure, or the addition to that science of some propositions not pertaining to it.

Purpose of this Section. — The following summaries, hypotheses, and theories are grouped here for two purposes: first, to show the general trend of speculation by the most eminent physicists, upon the nature of matter and of en-

ergy ; second, to indicate thus certain ways in which energy may be supposed to produce force or, in other words, to describe briefly certain mechanical operations which have been imagined as possibly representing that action of energy which we call force. This second point is really embraced in the first. The manner in which the expansive force of gases is explained, under the kinetic theory of gases, affords one such illustration. This is shown in another phase in the vortex-atom hypothesis. The latter theory gives a radically new view of the possible nature of chemical action. As a whole it also affords an unprecedented example of the deduction of most diverse laws and phenomena by pure mathematics, starting with a supposed material system of the utmost simplicity of character. No student of physics can afford to deny to these various hypotheses a most careful consideration. It is by far too customary to regard them slightingly because of the recognized improbability of their ultimate truth.

CHAPTER II.

KINETIC THEORY OF GASES.

Statement of the Theory. — If a perfectly elastic, spherical ball in rectilinear motion strikes against a fixed, perfectly elastic, plane, frictionless surface, it rebounds from that surface with unchanged velocity and in a direction in accordance with the law of reflection of light. The rebounding ball thus possesses precisely the same amount of kinetic energy as before impact. It has, therefore, imparted none of its energy to the surface with which it collided, but during the entire time of contact (this has a sensible duration) a stress existed between the ball and the surface. During the first half of this collision the kinetic energy of the ball is being transformed into elastic energy in the ball and in the surface. This energy is exerting a stress between the ball and the surface. During the second half of the collision the stress is maintained by their elastic energy, and the stress at any instant is equal to the space-rate at which energy is being interchanged between the two surfaces. Let us consider for simplicity the case where the ball strikes normally upon the surface.

If we were to consider the ball and surface merely as elastic substances without questioning the detail of the mechanism of this elasticity, we should describe a collision in this way: — As the ball comes into contact with the surface, its kinetic energy is gradually transformed into energy of elasticity within itself and within the surface with which it collides. This elastic energy is called into play by the change in form of the ball, which becomes flattened in the

direction of motion, and that of the surface, which becomes to a certain extent indented. During the time in which the ball is being brought to rest an increasing stress is exerted by this elastic energy between the ball and the surface. When the ball has come to rest, all this kinetic energy has been transformed. The stress between the surfaces and between the parts of the ball, as well as between the parts of the surface, is now everywhere at a maximum. The elastic energy producing this stress now sets the ball gradually in motion away from the surface, and the process is of such a nature that by the time all of the particles have returned to their normal positions the ball is again moving with its former linear velocity in all parts, and all parts of the surface have again come to rest in their normal position. Considering the force relative to the ball during the collision, it is a pressure *by* the ball during the first half and *upon* the ball during the second half of the impact. The bodies being supposed perfectly elastic, no energy remains in other forms.

To hold the surface in position against such a collision, it is clear that it must be supported by a force which at every instant is equal to the stress between the surface and the ball; otherwise the surface will move. This supporting force must evidently be in the same direction throughout the contact. If the collision is in a direction normal to the surface, then the supporting force is also normal to the surface and towards the ball. If the collision is oblique, the supporting force is still in the same direction as before, unless there is friction between the points of contact, because the component of the ball's velocity parallel to the surface remains unchanged by the collision, and the force exerted by the ball upon the surface has, therefore, no component in a direction at right angles to the normal. The stress between the ball and the surface varies from instant to instant, so that the supporting pressure will vary in like measure. We thus see how a spherical, elastic ball

entering into collision with a plane, solid, elastic surface produces a stress in the direction normal to it. If such a surface be supported by the hand, we shall experience a muscular sensation corresponding to the muscular effort which we exert. In all such collisions there will be certain minor changes in distribution of energy which have not been indicated. In the normal collision, a wave of vibration energy of small intensity will be transmitted in all directions through the restraining surface from the point of contact, so that the rebounding ball will in fact not carry off its whole initial amount of kinetic energy, and moreover will possess a definite, periodic vibration, which will consist of a periodic lengthening and shortening of the diameter in the direction of motion, and an accompanying corresponding change in other diameters. The kinetic energy will be less after than before impact by the sum of the energy of vibration of the ball and the vibration energy left in the surface. If the collisions are oblique, the ball and surface will similarly acquire vibration; and if there is friction between the ball and surface during the time of collision, the ball will also acquire rotation, so that in addition to the energy of vibration there will be a certain amount of energy of rotation to be deducted from the kinetic energy of the returning ball. These secondary actions involve usually but a small portion of the energy of a single impact, and their effect will be further considered in dealing with the phenomena produced by the collision of a large number of balls enclosed in an elastic receptacle.

Imagine now a continuous and uniformly distributed shower of such balls striking normally upon a small surface and rebounding without collision with each other. At any instant several or many of these balls will be in collision with the surface. But as any one is equally likely to be in any possible phase between the initial and the final state of the collision, a variety of phases will exist at any

given instant. As the density of such a shower is greater (that is, the number of particles impinging on a unit of area), more and more phases will be represented at any instant in the colliding balls; and ultimately all phases will be equally represented at every instant. Thus, the total pressure on the surface will be more and more nearly uniform the greater the density of the shower. If instead of the normal shower, the surface be struck by a shower of balls proceeding equally in all directions, oblique as well as normal, the stress will still be a normal one, and will, as before, be more nearly constant the greater the number of balls. If there is no friction between the balls and the surface, there will be no force in the plane of the surface; but if there be friction, the amount of such force in all directions will be equal, and its resultant will therefore be zero.

Imagine now a box of any size and shape with walls of perfectly elastic material and containing within it nothing (*i.e.* a perfect vacuum) except an immense number of exceedingly small, spherical, elastic balls such as have just been considered. Let these balls or particles be so minute that their number per cubic inch, if distributed uniformly through the volume of the box, would be reckoned by millions or billions, but let their size be so small that their total volume is exceedingly small compared to the capacity of the box; that is, suppose the capacity of the box many million times the total volume of the particles.

Imagine the box to be shaken for some time with great violence and then brought to rest, what will be the characteristics exhibited by the space thus filled with moving particles?

The motion of such an imaginary system has been mathematically investigated by Clausius, Maxwell, Lord Kelvin, Joule, Boltzmann and many others. It has been shown that the system of particles, whatever might be the initial

state of motion produced by the agitation above described, would ultimately and rather promptly settle down into a condition of kinetic equilibrium. This final condition at any point within the volume, and at any given instant of time, is such that a sensibly equal number of particles is moving in all directions. If the box and contents were at the same temperature, no kinetic energy would be given up to the box by the particles, but the total energy of the system of the particles would remain a constant, provided that the box transfers none of its elastic energy to surrounding bodies and that no energy of any other form were introduced through the box. The particles in their motion would, on the average, traverse in straight lines a distance very long in proportion to their diameters, but would be continually coming into collision with other particles or with the sides of the box. From these encounters with each other, the particles would rebound with changed velocities and directions, and therefore, in general, each with different kinetic energy, but the total energy of the colliding particles would be unaltered by the collision. The particles striking the side of the box would rebound, also with unchanged energy on the average, although in any individual collision there might be a greater or less change, owing to vibratory energy in the side of the box at point of contact. Each particle would possess not merely translatory motion but also rotation and vibration. Some of the particles at any given instant would be moving with very high velocity, some with very low velocity, and the larger proportion with intermediate velocities. Considering any one particle, the distance which it traverses between successive encounters would be sometimes long, sometimes very short; and the direction of its *free path* in these intervals would be changed at every collision. Its amount of energy of rotation and of vibration would likewise be changed at every collision. Although all these changes are continuously

going on in the individual particles, so that it is impossible to trace mathematically the history of the individual, still, Maxwell has shown how the particles might be treated by a statistical method, and has thus discussed the nature of the motion and has shown that the system would be governed by laws corresponding closely, in many cases exactly, to the known laws of gases. Boltzmann has shown by this method that a state of kinetic equilibrium will come about as above stated, in which in spite of the continual individual changes, the proportion between the total kinetic energy (heat), the total energy of vibration, and the total energy of rotation, in the system will be a constant. He has shown that if any disturbance of this proportion is introduced, a readjustment will spontaneously occur to bring back the former distribution of energy. This state of kinetic equilibrium is therefore a stable one.

It has been shown mathematically that such a system of particles would possess nearly, if not quite all, of the properties exhibited by a perfect gas. By their bombardment upon the sides of the box, the particles would produce a pressure in every way analogous to gaseous pressure. Across any imaginary plane surface, at any point within the gas, there will be a transfer of particles. If this plane were to become solid, the transfer would be changed to a bombardment which would produce upon either side of the plane a pressure of the same intensity as upon the walls of the box, thus illustrating the internal pressure of a gas. If the portions on either side of this imaginary plane were to move over one another, they would experience internal friction corresponding to the internal gaseous friction, or viscosity, described at page 49. Any one particle would move gradually and irregularly from one to another part of the box, and the nature and laws of this motion would correspond precisely with those of gaseous diffusion. The mean square of the velocity in such a system would be pro-

portionate to the temperature of the gas. And the laws of Boyle and Dalton follow directly as properties of the system. The pressure exerted by the system per unit of area of surface exposed would be normal to the surface and proportional to the average kinetic energy of the particles; and two such systems, each composed of particles of the same size and mass as those of the other system, or of different sizes or masses, would be in thermal equilibrium when the average kinetic energies of the particles of the two systems were equal. If two boxes containing each a distinct system were to be brought end to end, and the partitions removed so that the systems would be in direct contact, then, if the pressures were equal, there would be no resultant transfer of kinetic energy (that is, of heat) across the junction in either direction, provided also that the average kinetic energy of the particles of the two systems were the same. If the two systems were mixed together, the same relation would hold when the normal distribution of the particles was attained.

Maxwell has shown that this condition of equality of kinetic energy is the condition for temperature equilibrium, not merely between such systems as those above described which correspond to different gases, but also between *any* two molecular systems, as between solids and liquids, or either of these in contact with gases. This point seems to be one of fundamental importance, since without it there would remain to be shown a reason why the kinetic energy, that is, the heat of an enclosed gas, should not be carried away in the form of waves of vibration energy through the enclosing surfaces however perfect their elasticity. This point appears not to have been generally considered in dealing with the theory of gases.

The kinetic theory of gases is based on the assumption that the gas is constituted of perfectly elastic particles, whose dimensions are excessively small compared with the

volume of the gas, and whose free paths are very long compared with the diameter of the particles. In its simplest form it makes no further assumption regarding these particles than those which are contained or implied in the above-described system of moving spherical particles, and the nature of the particles of the gas is supposed to be similar in all respects to that of the system of elastic particles just described. Most of the properties of perfect gases can be accounted for by this hypothesis, and with a few additional hypotheses or considerations, the deviations of gases from the simple laws, as they approach their points of condensation, can be more or less fully accounted for. Some phenomena seem to call for additional hypotheses, ascribing to the particles forces of mutual attraction or repulsion, or both, varying according to arbitrarily assumed powers of the distance between the centers of the particles. Of course such an addition complicates the theory in a high degree, and these assumed forces in turn demand an explanation; still, even in this form, the kinetic theory has proved of the utmost value in advancing our knowledge, both theoretical and experimental, of the laws of gases. Many of the deviations, however, are accounted for much more simply and with a high degree of approximation by considering the influence upon the laws of the supposed system which would arise from assigning to the particle a greater size in proportion to their average distance apart.

From various data it has been found possible to calculate approximately the average velocity of the molecules of any gas at ordinary temperature, the mean length of free path, and very roughly the apparent diameter of the particles or molecules. In the article "Molecule" in the Encyclopedia Britannica, Maxwell has tabulated values for several gases. As belonging to a familiar gas and one which is a large constituent of air, the following values for oxygen are quoted.

Mass of Molecule (hydrogen = 1) = 16.

Velocity (of mean square) at 32° F. = 465 meters per second (= 18000. inches or 1500. feet per second).

Length of mean free path = . 00 000 0056 meters (= 0.00 000 22 inches).

Collisions of an average particle per second = 7 600 000 000.

Diameter = 0.00 000 000 076 meters (= 0.00 000 0030 inches).

Mass of Molecules = 0.00 000 000 000 000 000 000 0074 grammes (= 0.00 000 000 000 000 000 000 0005 grains).

Lord Kelvin has discussed the conditions of equilibrium, and the various properties, of systems of such particles grouped in more or less complicated ways. He has investigated the condition of stability of such groups, and of mixtures of groups of different complexities, the individual particles of the group being held together by mutual forces. He has thus been able to arrive at systems which would exhibit many of the peculiar properties, both mechanical and electrical, of liquids and even of solids. He has also been able to find analogies for the case of a liquid or solid in contact with its own vapor.

Many other attempts have been made to extend these kinetic explanations of the structure of solids and liquids, but the difficulties are here much greater than for gases, owing to the close proximity existing between the particles, and to the play of the mutual forces, which is negligible in gases because of the great distance between the particles.

An excellent presentation of the kinetic theory of gases may be found in Maxwell's "Theory of Heat," Watson's "Kinetic Theory of Gases," and Risteen's "Kinetic Theory of Gases." No one who wishes to follow out the literature of the subject should fail to read the various contributions to it by Maxwell ("Scientific Papers" and his articles in the Encyclopedia Britannica), by Lord Kelvin ("Mathe-

matical and Physical Papers"), and by Clausius (articles in Poggendorff's "Annalen").

Comment. — Of the kinetic theory of gases, and of the corresponding molecular theories of liquids and solids, it may be said that they have served and continue to serve a most useful purpose in the true function of theory. They are, however, not ultimate in character. The assumed perfectly elastic molecule, with its attendant supposed forces and properties, plays satisfactorily the rôle of a unit in a great company whose effect as a whole only is to be considered. To some extent it is effective when treated as an individual also. But as far as any ultimate explanation or even simplification of the forces, forms of energy, or views of matter are concerned, it is of small effect. The molecule, as its name implies, is merely a little *body*. The molecular theories offer no account of energy, of elasticity, or of the energy which must be present in the molecule or atom to give it shape, hardness, etc. It is left for the vortex-atom theory to take the first step toward the ultimate analysis of the properties of substance.

CHAPTER III.

LE SAGE'S THEORY OF GRAVITATION.

Statement. — In the year 1782 Le Sage of Geneva published a hypothesis or theory, to explain the action of gravitation. Somewhat similar, but much less complete and careful explanations had been previously suggested, and to them Le Sage makes conscientious reference. The theory has been modified by others, but a sketch of it in the original form, omitting for brevity some of the limitations which Le Sage so carefully stated, will be here given.

Imagine an indefinitely large space containing nothing except streams of minute particles (or as Le Sage calls them "corpuscules") flowing through the space in a multitude of directions. Each stream consists of particles flowing parallel to each other, uniformly distributed, having an enormous velocity. Each stream is supposed to have a sectional area sufficient to cover all parts of the space under consideration. The directions of the various streams are supposed to be uniformly distributed, so that all directions are equally well represented. The particles are supposed to be *perfectly hard and inelastic*, and of a size so small relatively to the space between them that collisions between the particles of the various intersecting streams are exceedingly rare — a condition which is aided by the enormous velocity ascribed to the particles.

At any point within this space, then, imagine a small sphere to be described. This sphere would be bombarded equally on all sides by the corpuscules. If the sphere were permeable by the particles, they would simply pass

through it. If it were wholly impermeable, they would either be brought to rest by it, if the sphere were inelastic, or they would rebound from it if the sphere were perfectly elastic. In either case, the sphere would experience an equal pressure from all sides, and would therefore be in equilibrium in the midst of the moving particles.

Suppose now a second sphere near the first. If both were entirely permeable they would be in equilibrium, and no motion of either would ensue. If, however, they were wholly impermeable, then each would arrest the particles coming against it from the side turned away from the other sphere, and each would thus cast a sort of shadow upon the other. Thus sphere number one would receive less bombardment on the side next to number two, and similarly number two would have less bombardment on the side towards number one. The pressure, therefore, upon the sides facing each other would be less than the pressure on the outward-facing side, and each sphere would, therefore, experience a force whose direction it is easy to see would be towards the other sphere; in other words, the spheres would "tend to approach" one another.

Le Sage assumed the corpuscules to be perfectly inelastic. Suppose also, for the moment, that the spheres are perfectly impermeable. It will be clear, on a little reflection, that the amount of the force upon each sphere will be, in this case, proportional to the product of the equatorial sections of the two spheres. For the moment, consider each sphere to be replaced by an impermeable, circular, plane surface whose diameter is equal to the diameter of the corresponding sphere, and whose plane is perpendicular to the line joining the centers of the two spheres; then circle number one will cut off from each unit area of the circle number two a definite fraction of all the particles which would otherwise strike it from that side. This fraction would be the quotient of the apparent area of the small circle as seen

from the unit area considered, divided by the total area of the hemisphere whose radius is equal to the distance between the two circles. The pressure per unit area of circle number two will, therefore, be less on that side by a definite amount, which is proportional to the area of circle number one; hence the whole pressure on circle number two, being the pressure per unit of area multiplied by the total area, will be less than that on the other side by an amount which is proportional to the product of the areas of the two circles. This is only approximately true when the circles are of a diameter which is not small compared to their distance apart.

Considering any point on the circle number two, it will be seen that the apparent area of circle number one, as viewed from that point, will be inversely as the square of the distance between the circles. The fractional part of the total pressure upon circle number two cut off by number one will, therefore, be inversely as the square of the distance between the circles. This is also only approximate, but more nearly so in proportion as the distance between the circles is greater in proportion to their diameter.

Replacing the circles now by the original spheres, it is evident that the pressures will be the same as before, and each sphere will therefore experience a force towards the other which is proportional directly to the product of the equatorial cross-sections of the spheres, and inversely as the square of the distance between their centers.

If the spheres are not wholly impermeable, but are partially permeable, then each will cut off from the other only a portion of the streams of corpuscules. The law as to distance will clearly remain unchanged, but the amount of the force will now depend not merely upon the equatorial cross-section of the sphere, but upon the structure of the sphere. If we imagine a sphere to be made up of separate particles with considerable space between them, the particles them-

selves being impervious to the corpuscules, then the amount of the screening by the sphere as a whole will depend on the disposition and size of its particles. Eventually, particles nearer the surface will tend to screen those below the surface, so that the total screening will not be directly in proportion to the total number of particles composing the sphere and, therefore, to its mass. In order that all of the particles shall be equally exposed to the bombardment of the corpuscules, or nearly enough so, it is necessary to devise some open structure for the material of the body. This Le Sage did by imagining the particles of the sphere to be not spherical or compact particles, but to have their material distributed in lines along the edge of open cubes or other geometrical figures. This structure gives to each particle a greater surface exposure in proportion to its mass, and still leaves a very open structure of the whole sphere, so that the corpuscules may more easily penetrate it. By imagining this openwork structure carried to the necessary extent, and by assigning to the corpuscules any necessary degree of minuteness, the screening can be made as nearly as desired in direct proportion to the mass of the sphere. It is essential that the structure should be so open and the corpuscules so minute that by far the greater part of them pass freely through the material bodies, even when they are of the size of the sun or other celestial masses; because there is no evidence that the action of gravitation between any two such bodies is lessened when a third similar body is interposed. This is shown on a large scale in the case of the attraction between the sun and the moon at the time when these and the earth lie nearly, but not quite, in the same straight line as compared with the time when they lie quite in the same straight line so that the moon is eclipsed.

Imagine, then, two such spheres, and we shall have the forces upon each proportional to the product of the respective masses, and inversely as the square of the distance of

their centers, as is demanded by the observed (Newtonian) law of gravitation; moreover, the approximation to this law is no longer the rough one holding for the parallel circles, but is a much closer one, depending only on the degree of fineness of the corpuscules and on the openness of the structure of the sphere. The observed law of gravitation can be also easily demonstrated for bodies other than spheres, these having been taken merely for simplicity.

What has been shown for two spheres may be extended to the mutual relation of any number of gravitating bodies, and we then have a mechanical explanation of the cause of the force of gravity substantially as given by Le Sage.

Objections.—In order that the Le Sage system of bombarding corpuscules should produce a resultant pressure as supposed, when acting on two neighboring bodies, the corpuscules must give up all or a part of their translatory energy at impact with the molecules of the bodies. Otherwise the corpuscules rebounding from the near sides of the bodies will exactly replace those screened out by the bodies, and there will be no "shadow" or resultant force. The corpuscules, being inelastic, cannot acquire vibration through impact, neither can they receive rotation unless friction be presupposed, so that without loss of energy they cannot lose translatory velocity, and, therefore, cannot cause gravitation. But the force of gravitation to be explained exists not only when the bodies are in approach but equally when they are at rest or receding from each other. Thus it is requisite that the corpuscules shall be yielding energy to the bodies at all times. Also, as there can be no discrimination, this must be true of every impinging corpuscule. What becomes of all this immense quantity of energy beyond that part used in such bodies as may be approaching one another? And what is the source of this energy? These two queries indicate difficulties which are fatal to the Le Sage theory in its original form, or at least to the inelastic corpuscule which Le

Sage invokes. No theory is tenable which, to explain the limited phenomena of gravitation, postulates a disproportionate rate of expenditure of energy and one which must be perpetually maintained from an unknown source. Add to this that the stream of energy must be inevitably transformed directly into heat, and one might suppose the theory beyond hope of resuscitation. But good reason for its revival seems to be supplied by the vortex-atom theory to be developed.

We will here however review some obvious modifications of the Le Sage theory. Gravitation seems to have to do directly with the atom rather than with the molecule. All considerations appear to impose perfect elasticity upon the atom. Adopting this view, what would be the result of assuming the Le Sage corpuscules to be also perfectly elastic? If no vibration or rotation of the corpuscules resulted from their impacts on the atoms, the system would be impotent to produce gravitation. The case would be like the invisibility of a body entirely enclosed by surroundings all at the same temperature. There would be no corpuscular shadow between bodies. The sum of the transmitted and reflected corpuscules would be the same in the same direction at all points. But as the corpuscules and atoms are elastic, each collision will be attended by a change in amount of energy of vibration in both corpuscule and atom. Also if there is any friction at the points of impact, or if the corpuscule is not spherical, there will likewise be a change in energy of rotation of the corpuscule. A Le Sage system of elastic corpuscules devoid initially of vibration and rotation would be capable of producing the gravitation shadow. For the rebounding corpuscules on the near sides of two bodies would have less translatory velocity than the impinging corpuscules on the far sides, whose places they should make good if the shadow is to be prevented. In other words, the average translatory energy of the corpuscules is less between

two bodies than on the far sides, the ultimate reason being the conversion of some linear into vibration or rotation energy. At this point enters the objection raised by Lord Kelvin but afterwards withdrawn: namely, that the translatory energy of the system would be continually and rapidly frittered away into distributed and unavailable energy of vibration and rotation; so that a continual and enormous supply of energy would be needed to maintain the force of gravitation even when no motion ensued. This otherwise formidable objection is dispelled by Boltzmann's compensation proposition stated at page 177. This shows that in such a system the vibration and rotation energies would each approach a definite ratio to the translatory; and that if this proportion were then in any way disturbed by external action, the system would automatically proceed to restore the balance. No energy therefore is consumed in the continuous operation of such a system except in the actual performance of gravitation work. And the quantities demanded for the performance of even so enormous pieces of work as the approach of the planets to the sun in their orbits is so utterly insignificant as compared with the whole energy of the system of corpuscules, that its abstraction can produce no sensible lessening of the force of gravitation.

A more serious difficulty arises from the consideration of temperature equilibrium, as pointed out by Maxwell. In the entire system of corpuscules and atoms, the condition of temperature equilibrium is that the average kinetic energy of the corpuscules shall be equal to that of the atoms. Now in order to provide for the enormously long free path of the corpuscules it would be natural to assume the number of them in a unit of volume to be small, each one being of the requisite mass. This would make the frequency of collision smaller and the free path greater. But Maxwell asserted that under this condition the corpuscules would raise the temperature of the atoms to white heat.

In other words, the kinetic energy of the corpuscules would be far in excess of that of the atoms, so that bodies would be intensely heated. It has, however, been pointed out in rejoinder that the assumption on which this objection rests is apparently not an essential one, and that we are at liberty to assume a large number of corpuscules per unit volume if the temperature conditions can thereby be met; for the three variables, mass, number, and velocity, are mutually adjustable to meet the facts which the hypothesis has to explain. By increasing sufficiently the number and velocity of the corpuscules, and therefore diminishing their size, there appears to be no intrinsic difficulty in providing the necessary free path, and at the same time providing temperature balance. At least, the contrary seems not yet to have been shown.

It appears then that no objection has been advanced which conclusively discredits the competence of a modified Le Sage system to furnish a mechanical analogy to gravitation. The elements and conditions of such a system would be substantially these. Consider a space of stated dimensions, throughout which gravitation is to be maintained. Imagine a myriad of perfectly elastic particles or corpuscules moving through this space in such a manner that all directions are equally represented in the motion, that is, that a sensibly equal number of corpuscules will be passing in every direction through a small sphere located at any point of the space. The corpuscules must be exceedingly minute as compared even with the atom of ordinary bodies. Their mean velocity must be many times that of light; and their number per unit of volume, great as compared with the number of atoms even in solids. The mean free path of the corpuscules between collisions must be many times the largest dimension of the prescribed space. Finally, the system of corpuscules must be either of infinite extent or surrounded by a perfectly elastic adiabatic boun-

dary, that is, one which transmits no energy to or from the system. The quantitative relation between the mass of a corpuscule, the number per unit of volume, and the velocity, is conditioned by the requirement of temperature equilibrium between the system and material bodies. As the temperature of all bodies is not the same, and as exact equilibrium at one temperature will involve the heating of bodies at a lower temperature, and the continual cooling of those at a higher one, we discover a point of interest which seems not to have been brought forward. To assign to the corpuscular system the temperature of absolute zero would involve a contradiction, since the system is based on the kinetic theory of gases in which temperature is proportional to the mean square of the velocity of the particles in a given gas, and the corpuscules must of course be in motion. On the other hand, space appears to be very cold, so that the corpuscular temperature must be low. If then gravitation is due to such a system, it must be accompanied by a continual cooling of nearly or quite all bodies. But whether or not we adopt this last inference, we can hardly escape the more general one, that some temperature must be assigned to the system, and that bodies will therefore be heated or cooled by the system according as they are above or below that point. Have we here to do with an insurmountable obstacle? There is certainly in operation no known process of heat transfer with laws such as would arise from this. The well-known thermal radiation into space is a surface phenomenon, at least as contrasted with this one which would proceed with equal facility at all points throughout a body however large. This is sufficiently serious, but on the other hand the fact must be recognized that the whole phenomenon viewed as a case of the interaction of two very diverse sets of elastic particles specially conditioned, has not received adequate mathematical investigation. Something not indicated in the

kinetic theory of ordinary gases may be anticipated from a rigid extension of that theory to meet the condition of the joint system of corpuscules and atoms assumed to occur within bodies. The enormous disparity between atomic and corpuscular size, free path, velocity, and number per unit of volume, form one ground for such anticipation. Another is afforded by the entanglement of corpuscules within the bodies with which they collide. This must occur to a considerable extent because the gravitation shadow is due to the transmutation of linear into rotational or vibrational motion in the corpuscules, and further because this change occurs not only in the relatively small portion of the corpuscules engaged in producing a shadow, but in every colliding corpuscule. Mathematical investigation alone can clear up these points.

In the problem of temperature equilibrium, then, appears to lie the crux of the Le Sage hypothesis. The forecast is certainly not favorable, but there is still a future for the theory. No other affords so comprehensive an analogy to gravitation, and none is so helpful to the student who finds his imagination unequal to coping with a purely abstract notion of this fundamental property of bodies.

Two points in the theory call for further notice; the first of these is the permeability of bodies to the action of gravitation. Le Sage was forced to meet this by a bold assumption that the atoms are of an open structure as explained at page 185. This, apparently not being demanded or justified by other physical facts, has undoubtedly been a great incubus to the theory. One cannot therefore refrain from a feeling of pleasurable surprise at the manner in which the vortex-atom removes this difficulty. To the hypothesis, objection has also been raised on the score of the enormously great velocities, great compared even with the velocity of light, which it is necessary to ascribe to the corpuscules; and on the score of the extraordinary minute-

ness necessary to be assigned to the corpuscules; also of the great length of free path between collisions. To these it may be fairly answered that no objection upon the ground of mere minuteness or immensity is really tenable. A thing is merely great or small relatively to what we have been accustomed to deal with, and there is no absolute scale of magnitude. As to the length of free path, it is only necessary that this should be assumed long as compared to the dimensions of the solar system, or of such groups of celestial bodies as double stars, etc., in which alone we have evidence of the existence of gravitation. We are not called upon by observed facts to account for any law of gravitation through spaces greater than these, since the facts do not show that such gravitation exists.

It is to be noted that the Le Sage bombardment theory yields only an approximation to Newton's law of mass and distance. For distances large relatively to atomic dimensions, the approximation may be rendered as close as desired by the assignment of proper corpuscular dimensions and velocities. And this, it must be remembered, is all that is called for by the facts. For our experimental knowledge of the law is accurate only within limits, say to one part in one million. Moreover, at short distances there is no evidence that the force between molecules is exactly that which would be indicated by Newton's law. There is thus good reason to suppose that the law to be accounted for may not be a constant one, but may change with the distance.

CHAPTER IV.

THE VORTEX-ATOM THEORY.

Vortex Motion. — This theory of the constitution of substances was suggested by Lord Kelvin soon after the publication in 1858 of Helmholtz's very remarkable mathematical discussion of the possible rotational motion in a perfect fluid. The theory has been developed from time to time both mathematically and in its applications, particularly by Lord Kelvin and Professor J. J. Thomson (cf. Helmholtz's Scientific Papers; Thomson, Sir William, Proc. Roy. Soc. Edinb. 1867, also "Nature," XXIV., 47; "Mathematical and Physical Papers"; J. J. Thomson's "Motion of Vortex Rings," Macmillan & Co., 1883; Maxwell, Encycl. Brit., article "Atom"). The following sketch of this theory is designed to give a general view of its scope and basis, not to present a mathematical, exact, or complete statement of it, the latter indeed being impossible without resort to a highly complex mathematical treatment.

The theory undertakes to deduce by rigid mathematical demonstration analogies to the properties and laws, both physical and chemical, of material bodies, from the simple laws of motion of an ideal perfect fluid. The purely mathematical difficulties in the treatment of this subject have restricted the progress of the hypothesis thus far chiefly to the physical laws of gases, but a surprising extension of it into the domain of chemical phenomena has been made by J. J. Thomson.

It is essential first to define what is meant by a "perfect fluid," and to examine the properties ascribed to it. The fluid whose motion is discussed is assumed to be "material"

in the sense that it possesses kinergety, the capacity for kinetic energy; or some property from which kinergety may arise. It is further assumed to possess inertia, to be continuous in space, homogeneous, absolutely incompressible, and frictionless (that is, devoid of internal friction). If a limited portion of the fluid is dealt with, it must be supposed to be surrounded by a wall or boundary which is perfectly rigid, so that whatever motion of the fluid takes place towards the boundary, no displacement of that boundary occurs. If we deal with an infinite volume of the fluid, no boundary is requisite, because the infinite kinergety of the fluid with reference to any point within the infinite volume is equivalent thereto. No other attributes or conditions than these are ascribed to the fluid in any part of the hypothesis or its development. It is important to scrutinize these attributes in detail. In doing so we are at once impressed by the fact that they are *all negative in character* except kinergety; that is, that none of them requires the assumption of the existence of energy of any kind, as would the property of elasticity, or the assignment of forces to any portion of the fluid or to its boundary. Thus inertia (cf. p. 19) is merely the denial of any power of change of motion; unless indeed the term was employed to denote kinergety, — in which case it brings no further attribute than has already been imposed by that term. Inertia is properly only a denial of energy. Homogeneity is merely the denial of any structural differences in the fluid. Continuity is merely the denial of any break in the fluid. It may be noted that in a finite mass without a boundary, continuity could not be maintained, if motion of any portion of the fluid took place, without the existence of a force of cohesion or its equivalent, implying energy; hence, the necessity for the assumption of a boundary or an infinite volume of fluid. Finally, absence of friction is merely the denial of the existence of any force of friction, or any kind of action, within the fluid which would

give rise to such a force when relative motion of parts of the fluid occurred. In the section on "Resistance" it is shown that frictional resistance of all kinds implies the existence of some form of energy; hence the denial of friction is merely another form of denial of the existence of energy in the fluid. Incompressibility appears (at least to the author) to be simply a denial both of elasticity and of discontinuity; for it is not conceivable that the total volume of a given portion of a substance can be lessened unless it consists of distinct portions with unoccupied space between them, or unless the continuous material has ascribed to it the property of elasticity. It may be remarked that this assertion of incompressibility is equivalent to an assertion that matter occupies space, and leaves this property of occupying space which is identical with incompressibility, as a fundamental property of matter.

This hypothetical perfect fluid is, then, what this theory presents as *matter* (or more properly as its analogue), using this term in the sense in which it is defined in this book and in which Lord Kelvin appears to employ it when he chooses to discriminate between matter and material substances or bodies.

Any finite portion of an infinite volume of a perfect fluid may have either or both of two modes of motion: namely, first, translatory or irrotational motion; second, rotational or rotary motion. The second kind of motion exists in any portion of the fluid when, if that portion be imagined to be suddenly solidified, the solid would be in rotation about some axis passing through it. The first kind of motion occurs when any portion of the fluid is moving in a manner such that if suddenly solidified the solid would show no rotation about any axis. Lagrange, Euler, and other mathematicians had investigated the irrotational motion of such fluids, but the rotational motion had not been discussed until Helmholtz published his paper on

"Vortex Motion." In this he deduced the equations of motion of rotation as well as of translation in a perfect fluid. This was purely mathematical discussion with no reference to the theory of the nature of matter. With the irrotational motion we are not particularly concerned.

The discussion of the rotational motion led to very remarkable and novel conclusions.

The first of these to be stated should perhaps be, that if rotational motion has been imparted to any portion of the fluid, that motion can never be transferred to any other portion of the fluid, nor be changed into irrotational motion. Thus any portion of the fluid once in rotation remains permanently in rotation.

It is also shown that the translatory motion of any portion of the fluid cannot be transformed into rotational motion; this, however, had been previously demonstrated.

Thus rotational motion can never be produced or destroyed in any portion of such a fluid and has therefore the character of entire permanence. Energy of translatory motion can be transferred from one portion to another but, under the principle of the conservation of energy, cannot be destroyed.

To picture the vortices, imagine a rubber rod of circular cross-section with a diameter small compared with its length and with its ends cut squarely off, to exist within such a perfect fluid. That is, suppose for a moment that it replaces its volume of the fluid. Imagine the rod to be in rotation about its central line, that is, about a central axis passing from one end to the other. Picture the motion of each portion of such a rod as it revolves. Now imagine that the rubber rod is replaced again by the fluid, and that all portions of the fluid thus introduced move precisely as the corresponding portion of the rubber rod previously moved. We shall have here a circular column of fluid revolving upon a central longitudinal axis. Now Helmholtz

showed that such motion can take place in only two cases: first, where the column ends only at the boundary of the liquid, that is, where each end extends to the boundary, whether this be finite or infinite. Second, where the column is of finite length, but where it has been bent around until the opposite ends have been squarely abutted against each other, thus forming a continuous ring instead of a column. The motion within such a ring may be pictured by imagining the rubber column as before in its motion of rotation around its longitudinal axis, and then imagining the column to be bent until the ends abut squarely, and supposing those ends to be cemented together, so that the ring becomes continuous. Each point in such a ring will then be revolving about the central line of the ring in a circular or nearly circular path whose plane is at right angles to the central line; and if we draw a radius of any such circle, all the particles once on that radius will continue forever to remain upon it as the radius revolves with the motion of the ring. In the supposed case, the ring will be circular. If this rubber ring be now replaced by fluid revolving in the same way, we have a revolving circular ring of fluid which will always consist of the same parts and will always retain its motion. Such a column, and such a ring, of rotating fluid are called a *vortex* and a *vortex ring* respectively. The central axis of rotation is called the vortex line. In the case of the supposed ring this is a circle whose diameter is the mean diameter of the whole ring. Such a vortex ring once set in rotation is, therefore, absolutely permanent in its existence and rotation, and by the same reasoning, no such ring could be started by any finite means, and could therefore exist only by "an act of creative power." It is with the motion of such rings that we are to be concerned, and not with the motion of the vortex column.

The "strength" of a vortex ring is measured by the prod-

uct of the cross-section of the ring into its angular velocity of rotation. This product is shown to remain constant for any given ring. Now as the total volume of such a ring also remains constant, it is clear that if the ring enlarges in diameter, and consequently diminishes in cross-section, the angular velocity must increase in the same proportion. The increase in diameter, therefore, is attended with increase of angular velocity and diminution of cross-section; while a decrease of diameter is accompanied by the reverse changes.

Vortex rings in the perfect fluid may possess translatory motion and, therefore, the kinetic energy belonging to such motion of matter. As there is no friction in the fluid, there will be no retardation of the translatory motion of the ring from that cause, and as the continuity of the fluid is maintained and it is incompressible, there will be no excess of pressure upon the front of an advancing ring over the pressure upon the rear surface. That this should be the case implies as stated above the existence of a rigid boundary if a finite volume only of the fluid is dealt with, but if the volume is infinite, the infinite kinergety of the portion of the fluid towards which the vortex is moving replaces the effect of the boundary.

Rings similar in character to the vortex rings of a perfect fluid may be easily produced in air, and will exist there for sufficiently long periods for observation, although their properties and duration in the air are affected by the fact that this is not a perfectly frictionless fluid. Air rings may be seen issuing from the smoke-stack of a locomotive under certain conditions, being made evident either by the smoke or condensed steam which is associated with them; irregularities in the distribution of the smoke or steam show plainly the rotation in the cross-section of the ring. These air rings also sometimes possess a rotation along the vortex line around the center of the entire ring, a motion super-

posed upon the vortex motion proper. Similar rings may easily be produced for experimental purposes by means of a simple apparatus constructed as follows: a cubical box eighteen inches on each edge has one side removed; across this is stretched a piece of cloth or rubber, and in the opposite side is made a circular opening a few inches in diameter. If this box be filled with smoke, and a blow be sharply struck upon the flexible cloth side, a puff of air carrying smoke with it will be forced out of the circular opening, and a circular vortex ring will move onward through the air, its path and motion being made evident by the smoke. Some of the properties of vortex rings can be exhibited by means of these smoke rings. If two such boxes be prepared, the rings discharged from one can be made to approach those from the other, and they will be observed, when coming nearly into contact with each other, to exhibit elasticity, rebounding from one another with vibration precisely as we might imagine rotating rubber rings to do under the same circumstances. The peculiarities of motion when a smaller ring is made to pass through, or approach, a larger one face to face, either when both are moving in the same or opposite directions, may be studied. If one box contains smoke and the other only clear air, the effect of the action of the unseen ring upon the visible one is said to be very striking. For rendering the rings perceptible, it is convenient to employ instead of ordinary smoke a cloud of ammonium chloride. If ammonia vapor and the vapor from warm hydrochloric acid are led through separate openings into one side of the box, they will form a cloud of the white sal ammoniac dust in the box. Further interesting experiments have been described by Professor Ball ("Philosophical Magazine").

It is shown that the vortex ring in a perfect fluid possesses an elasticity of form precisely analogous to that of the smoke ring. If such a vortex ring be approached obliquely by another ring, a change of form will take place

in both rings and they will rebound or glance off from one another without coming to actual contact, and each after rebounding will, in general, be in a state of more or less violent, elastic vibration. This elasticity of form is due wholly to the motion in the ring. It is, therefore, permanent and perfect; and we have thus an object whose elasticity of form is explained by the mere motion of a portion of matter. This is the only means yet found of providing a purely kinetic explanation for elasticity. In the kinetic theory of gases, it will be remembered that in order to explain by a kinetic method elasticity of the gas as a whole, it is necessary to assume perfectly elastic particles or molecules, or to ascribe to these molecules possession of forces. Thus the ultimate explanation of elasticity was either merely deferred or transferred to something equally difficult of explanation.

This property of elasticity renders vibration possible not only through the change of form of the vortex ring as a whole, but also through the change of form of the cross-section of the ring. The periods of vibration will be dependent upon the dimensions of the ring and upon its rate of rotation. The possibility of such vibration at rates inherently dependent upon the ring itself is a point of utmost importance in the development of the theory, particularly in connection with the phenomena of radiation, a portion of which constitutes the domain of spectroscopy.

We have thus far considered merely single, plane, circular rings. There is however no reason why two such vortex rings might not exist linked into one another, as are the links of a chain; or indeed a greater number of rings might thus be linked, and the linkage might be of different degrees of complexity. Any such linked set of vortex rings would be a permanent structure incapable of separation of parts by any process whatever; for it is a fundamental property of these rings that no ring can cut through the material of

another, and that no ring can be in any way cut, since, as Tait puts it, "it would wriggle away from the knife." We have, thus, one means of producing units of differing degrees of complexity. Again a ring may be imagined which shall be knotted upon itself; such as might be produced by starting with our supposed rubber column, tying a knot in it of any form whatever, of any degree of complexity, and finally abutting and fastening together the free ends of the column. The motion of rotation in such a true vortex ring would be the motion which the parts of the rubber ring would possess if the column were supposed to continue in the rotation about its axis during and after the process of knotting and splicing. Such a knotted ring is a mathematical possibility, and its properties are determinate and are entirely analogous to those of the plane circular ring. For reasons just given, any such beknottedness is permanent; thus we have a second means of producing differences of character in rings.

Application to the Explanation of Matter. — The propositions above stated comprise some of those originally deduced by Helmholtz. Upon examining them, Lord Kelvin was at once led to suggest that these vortex rings in a perfect fluid afforded a new and most promising basis for a theory of matter and of the constitution of material bodies. The fundamental hypothesis as to the nature of the fluid is devoid of the embarrassing assumptions attending that of solid perfectly elastic or perfectly hard molecules. If we then regard this perfect fluid as matter, and vortex rings in the fluid as constituting the ultimate parts or atoms of bodies or substances, we have the elements of Lord Kelvin's vortex-atom hypothesis. The perfect fluid, although we have no tangible evidence of its existence, demands of us the assignment of only negative properties with the exception of kinergety, and is the simplest conception yet put forward. From this simple hypothesis, it remains necessary, in order to shape the complete theory, to show by strictly mathe-

matical processes that bodies composed of such atoms would have properties, and would be governed by laws, identical with those shown by experiment and observation to belong to bodies composing the physical universe. This deduction is attended with great mathematical difficulties and has on this account been carried to but a limited extent, as already stated, but with results which are certainly most remarkable. Before proceeding to consider some of these, it may be well to note that this vortex ring possesses the fundamental characteristics demanded of a material atom: it is permanent in mass and volume; it is indestructible; it possesses, through size, linkage, and beknottedness, the possibility of varied but permanent individual characteristics; it possesses perfect elasticity without the necessity of any assumption other than kinergety and motion.

Application to Gases. — Upon this hypothesis, a gas would consist of a portion of space filled with the perfect fluid in which were vortex rings of exceedingly minute diameter distributed at an average distance apart great as compared with their diameter, and having random motion in all directions with high velocities, probably somewhat as the molecules of a gas in the ordinary kinetic theory. The following remarks will apply, unless otherwise stated, merely to such a supposed gas.

The pressure of a gas consisting of vortex atoms is explained in a manner closely analogous to that employed in the familiar kinetic theory of gases, but of course with modifications imposed by the characteristics of vortex-ring motion. This explanation has been given by Professor Thomson on the pages of "Nature" above referred to, and is substantially as follows: —

First, suppose the gas to be enclosed in a cylindrical vessel, with plane ends at right angles to the axis of the cylinder. Imagine a vortex ring approaching one of these ends with its plane parallel to the surface. As the ring

comes into contact with the surface, it gradually expands and slowly spreads out over the surface until it reaches the walls of the cylinder. It will then crawl slowly backwards along the sides of the cylinder towards the other end. It will give up momentum during this contact with the end first considered, and will acquire momentum again on leaving that end, the two amounts being equal; it will, therefore, exert a pressure as it approaches and as it recedes from the end. Suppose next that we consider an indefinitely large and closed mass of the gas, and suppose a large number of vortex atoms constituting the gas to be moving about in this enclosure. At any given instant, a number of these will be approaching any surface of the boundary or any imaginary surface in the enclosure. Suppose the number thus approaching to be large per unit area of the surface, then as the rings approaching the surface gradually expand, they will encounter portions of the surrounding atoms instead of the sides of the cylinder, as in the case first supposed, and will tend to move backwards, preventing each other's expansion. There will be thus produced a more or less thick layer of tangled rings immediately upon and over the surface, and at the end of a certain time after the beginning of such an operation, a state of kinetic equilibrium will come about in which as many rings will be coming away from this layer with their original average diameters and velocities, as will be entering into it. The gas will thus be exerting a continuous and sensibly uniform pressure upon the surface. The condition of this layer is complicated by rings approaching it more or less obliquely, but it is not difficult to see, in a general way, that the resulting effect of this action is essentially the same as that of the bombardment by elastic spheres described in the kinetic theory of gases. This condensed layer at the boundary between a solid and a gas must play an important part in a discussion of the observed phenomena of surface condensation.

By a brief mathematical demonstration, Boyle's law of the compressibility of gases can be shown to hold for a substance thus constituted. The equation which represents this law contains a small term which shows a deviation from the law on the side of greater compressibility. This so far as it goes accounts for the observed deviation of ordinary gases from the law at ordinary pressures. It is to be noted that this term involves no new fundamental assumptions.

The relation between the kinetic energy of vortex rings and the absolute temperature of the gases composed of them as just described, appears not to have been worked out with any fulness. For a gas consisting of single vortex rings, the square of the velocity is not proportional to the temperature. As the temperature rises, the diameter of the ring increases and its velocity lessens, although the total momentum of the atom increases with the temperature. For such a gas, therefore, it appears that the temperature would be as some inverse function of the velocity rather than in proportion to it. With more complicated ring atoms, a different relation is indicated between temperature and velocity. These facts constitute a point of radical difference between the former kinetic theory of gases and the vortex-atom theory, and J. J. Thomson has suggested that upon this difference might be based a crucial test of the two theories. The phenomena of thermal effusion of gases is pointed out as one of those whose laws, as deduced from the two hypotheses, should be radically different. A test would thus be afforded by the comparison of the deductions from the respective hypotheses, with experimental observations, under corresponding conditions. No such tests have yet been carried out, and they are likely to be attended by serious difficulty, both mathematical and experimental. The fact that temperature is not proportional to the square of the velocity of the vortex atom will probably lead to a different condition for tempera-

ture equilibrium between different gases and between substances in general, from that pointed out in the kinetic theory of gases, namely, the quality of kinetic energy of the molecules. If so, this would probably have a very important bearing upon many applications of the hypothesis, notably in the explanation of gravitation, which will be later referred to.

Applications in Chemistry. — The extension of the vortex-atom hypothesis to the explanation of various chemical laws and phenomena has been chiefly made by Professor J. J. Thomson, and is mainly restricted to gases.

The starting-point is a system of vortex rings in which every individual ring has the same, or very nearly the same, strength whether it is a single ring or one which is in any way associated with other rings. In other words, the fundamental unit is the single ring always of a given strength. It is to be remembered that the volume and strength and kinergety of such a ring are invariable, but that the diameter of the ring varies with sundry conditions, such as temperature, etc., necessitating a corresponding change in either angular velocity or cross-section, in order to maintain constant strength. The further condition is imposed that the cross-section of the ring is always exceedingly small in proportion to the diameter of the ring.

Single rings of equal strength may differ from each other not only as thus shown, but may be intrinsically different in the following ways. Different rings may possess different volumes, and therefore different kinergeties, although of equal strength. This may come either through differences of cross-section or through beknottedness. The latter condition may naturally be supposed to account for characteristic differences other than those arising from mere differences of kinergety.

Besides the difference between single rings, further differences in the character of bodies may come about in either

of two ways. Two or more rings may be linked into one another; two or more rings may be associated together by rotation about one another. These two methods of combination afford the basis of a theory of chemical action, and will now be explained in more detail. They serve, with no additions to the fundamental hypothesis, to give very instructive explanations of chemical affinity, atomicity, valency, dissociation, and other chemical laws. The theoretical explanation of the constitution of the atom and of atomicity is based on the linkage of vortex rings. The simplest possible atom is supposed to consist of a single vortex ring of the most elemental form. This constitutes a monad atom, but of course monads may differ in characteristics, through differences in kinergety and beknottedness as just described.

More complicated atoms are supposed to consist of two or more single rings linked into one another. We have then to consider next the system of linkage and the resulting atoms.

We may imagine two or more rings linked together in a great variety of ways. Probably many of the systems thus produced will prove, on mathematical analysis, to be unstable assemblages. They could therefore not be supposed to be originally formed, or if formed they must either rearrange themselves into some stable position, or break up into other and stable groups. The simplest method of linkage which has thus far been shown to be stable, and which upon investigation seems to afford all the necessary complexity, is as follows. The statements will be made only for rings of equal strength, equal diameters (under the same condition), of a cross-section small compared with the diameter of the whole ring, and where the shortest distance between combined rings is small compared with the cross-section of the ring, and rings without knots. The extension to other cases has already been sufficiently indicated for the purpose in hand.

Suppose the rubber column above described (p. 196) to have, traced upon its surface, a spiral line like a screw thread, passing once or more times completely around the column in going from one end to the other. If, then, the column be bent around into a ring with its ends abutted, the two ends of this line will come together, and we shall have a spiral line around a circular ring. Imagine that line to be a vortex ring of small cross-section. We shall thus have a spiral vortex lying upon the surface of a circular ring. Again suppose the column to be straight, and that two such spiral lines be drawn about it everywhere equidistant from one another. This will be like a two-threaded screw. Suppose the spirals each to make a whole number of turns in passing from one end of the column to the other. Imagine the column then bent around into a circle and the ends abutted. One end of each spiral will thus meet with the other end of the same spiral. If, then, we imagine each of these spirals to be vortex rings, we shall have two spiral vortex rings lying on the surface of a circular ring. Suppose the rubber now to disappear and to be replaced by non-rotating fluid. We shall evidently have two spiral vortex rings linked into one another in this special manner. The rings will be everywhere equidistant from one another as measured by the shortest line between them, and it may be shown mathematically that such a pair of linked rings is a stable arrangement. That is, if any part of either vortex be slightly displaced, the system will tend to return to its initial condition. This is the supposed constitution of a dyad atom.

Instead of two spiral rings corresponding to a two-threaded screw, we may have three, four, five, or six, arranged in precisely the same way. Any of these systems of linked vortices will be stable, but any system of seven or more vortices is shown to be unstable, a fact of great importance in connection with chemical theory. The triad, tetrad, pentad,

and hexad atoms are supposed to be thus constituted; and under this hypothesis no higher atomicity is possible.

This system of linkage may be slightly modified as follows. Suppose, in drawing two spiral lines around the rubber column, that instead of making a whole number of turns, each line makes an extra half-turn about the column. Then, when the ends of the column are abutted, the beginning of one spiral line will join the end of the other spiral line and *vice versa;* so that if now we start at any point, upon either of the lines, and follow it over the ring, we shall find that the two spirals are continuous with one another. That is, instead of having two separate linked spirals, we have one continuous spiral looped into itself. The properties of such a spiral, or rather of the corresponding looped vortex ring, are shown to be substantially the same as of the two spiral linked vortices. Similar description would show how we may have continuous linked vortices corresponding to three, four, five, or six separate linked vortices.

It is not denied that there may be other methods of linkage which will be stable, but no others are obvious and none have yet been discussed with any considerable fulness. As has already been noted, any one or more of the component spirals in the complex vortices may be in itself complex.

The theoretical explanation of chemical combination, that is, the union of atoms to form molecules, either elementary or compound, is based upon the association of rings through the process of rotation about one another.

If two rings approach one another in any direction and in any relative position, they will in general collide or come nearly into collision and will rebound from one another. The respective velocities and direction of motion will be changed by the collision, and more or less change in their rates and periods of vibration will occur. But under cer-

tain circumstances, the two rings will not separate but will remain associated in rotation about one another. The following is an illustration.

Consider two simple, circular, vortex rings. If they approach each other, moving in the same direction, with their planes parallel or nearly so, and their centers very nearly in the same line perpendicular to their planes, then if the rear ring overtakes the forward one, it will remain associated with it instead of passing away from it, under certain conditions. These conditions are that the strength of the two rings shall be very nearly equal, and that the difference in diameter of the two shall be small in proportion to the diameter of either. The two rings thus meeting will continue to revolve about one another in a way which may be thus described. Suppose the two rings to be of exactly equal strength; to have their planes parallel; and suppose that the rear ring is slightly smaller than the forward ring. When they reach the same plane, suppose their centers to coincide, and a circle to be drawn midway between the two rings. Instead of separating, the rings will remain associated, revolving about this circle, until some external disturbing cause forces them asunder. If the rings are nearly but not exactly concentric, and nearly but not exactly of equal strengths, they will similarly remain associated, revolving about a circle lying between them, and located so that its distance from each is inversely proportional to the strength of the ring. The shortest perpendicular distance between the rings will remain constant and the pair of rings will continue to move forward. It is only when the diameters are within a certain limit of each other that the rings are capable of remaining associated at all. The more nearly equal the strengths of the rings, and the more nearly equal the two diameters, the greater will be the disturbance requisite to cause their separation. We have thus the explanation of chemical combination between two atoms, either of

the same or of different kinds; also a clew to the reason underlying the relative stability of such compounds.

If three or more rings are brought together at once, or in succession, in a manner analogous to that just described for two rings, they will all remain together and revolve about a common line in a manner similar to the two rings, provided that similar conditions are fulfilled. But the chance of fulfilment of such a combination of conditions is much smaller for an assemblage of three rings than for the combination of two rings. Also for equal conditions, the stability of the combination is less; so that we should expect to find very many more molecules of two rings than of three, or four, or more.

Each of these two or more rings thus associated with each other to form a molecule may be called a primary of the molecule. These primaries, in order that the molecule should be permanent, must be of equal or very nearly equal strengths. It is very important to fix clearly in mind two points: no molecule can be stable whose primaries are not very nearly equal in strength, and no molecule composed of more than six primaries will be stable.

Any primary may be either simple or complex, but if it is complex it consists of two or more associated rings. In this case each of the rings associated to form the primary is called a secondary. These secondaries may be associated in either of two manners. First, there may be two or more independent rings rotating about each other in the manner described, and thus constituting a molecule. Second, there may be two or more rings linked together as above detailed, in which case the secondaries are permanently associated and therefore constitute more or less complex atoms.

Chemical combination may be regarded somewhat in this light. Suppose a limited space to contain a large number of atoms of all degrees of complexity, made up as above described. These would then represent atoms of all elements;

and it may be here noted that while we have but seventy different kinds of atoms to be accounted for, there is an almost infinite range of possibility in the complexity of vortices. At the same time the atomicity of the vortex atom is limited to six, — a deduction which strikingly tallies with chemical facts. Suppose the atoms to be small compared with the total volume of the space, and that they are all moving freely about in all directions. In a multitude of collisions which will take place, the conditions necessary for the combination of rings above indicated will be frequently fulfilled. That is, two atoms of equal strengths and nearly equal diameters will come into collision in such a manner as to remain rotating about one another. If we note that, as already stated, the unit, single ring in the entire system is of the same strength, and that the strengths of linked rings is the sum of the strengths of the component rings, we shall see that the dyad atoms have double the strength of the monad atoms, and so on. Thus the simplest combinations would be pairs of monads, pairs of dyads, and so on. Thus the first molecules to be formed, and the most stable ones, would be diatomic molecules of monad elements. Of these the most stable compounds would apparently be those whose atomic strengths were the most nearly equal, and whose atoms were the simplest. Combinations of two dyad atoms to form a molecule would probably be less frequent and less stable, since for these the conditions are slightly more complex. A similar line of remarks applies to the triads, and so on. Combinations of three or more monad atoms to form a molecule would be much less frequent, and similarly of three or more atoms of higher atomicity.

A single monad atom could not form a stable compound with a dyad atom or with any atom or molecule more complex than a single monad.

A dyad atom may combine with any other dyad atom.

This would constitute a molecule having two primaries, each primary being a dyad, that is, having two linked vortex rings.

Or a dyad atom may combine with a primary consisting of two monad atoms associated in rotation; that is, the dyad may combine with a molecule consisting of two monad atoms. It cannot combine alone with an atom of higher atomicity or with a primary of more than two monads, on account of inequality of strength.

Or a dyad may combine with two other primaries, which may be, respectively, both dyad atoms; both, molecules consisting of two monads; or one, a dyad atom, the other a molecule of two monads. This molecule of three primaries has less stability than the preceding one of two primaries.

We may proceed in a similar manner, under the condition that all primaries must be of equal strength, to demonstrate what atomic combinations into molecules are possible or impossible. For instance, a triad atom could not form a stable molecule with a dyad atom or with a dyad atom and a monad atom. The tetrad atom could not combine with a triad atom, nor with a triad and a monad, nor even with two dyads, unless they were associated as a pair in rotation.

These illustrations will suffice to show how readily the hypothesis accounts for atomicity, quantivalence, and chemical combination in general. We may however note that it has not yet accounted for the changes of energy attending chemical actions, which shows itself so largely in the production or absorption of heat and electrical energy. We will now apply the hypothesis to certain typical chemical combinations. From the usual considerations and some others based upon vapor-density determinations and so on. Professor Thomson, in his book above referred to, has classified the elements provisionally according to their

atomicity, valency, or quantivalence. His classification is only partial and is not to be regarded as a final one, not only because of the difficulty attending such a classification ordinarily, but from certain added ones which the novelty of this hypothesis naturally induced. According to his grouping:—

The monads or univalent elements consist of:—

Arsenic, Bromine, Chlorine, Fluorine, Hydrogen, Iodine, Mercury, Nitrogen, Phosphorus, Potassium, Rubidium, and Thallium.

The dyads or divalent elements are:—

Cadmium, Carbon, Chromium, Copper, Lead, Manganese, Mercury, Oxygen, Selenium, Sulphur, Tellurium, Zinc.

The triads or trivalent elements are:—

Aluminum, Antimony, Bismuth, Boron, Indium.

The tetrads or quadrivalent elements are:—

Silicon, Tin.

Thus the atom of hydrogen consists of a single vortex ring; the atom of oxygen of two linked vortex rings; the atom of aluminum of three linked rings; the atom of silicon of four linked vortex rings.

The molecule of hydrogen consists of two hydrogen atoms rotating around one another. The molecule of oxygen consists of two atoms of oxygen rotating around one another, that is of two pairs of linked vortices, each pair rotating around the other, and so on.

The molecule of hydrochloric acid, HCL, consists of two primaries associated in rotation, each primary being a single vortex ring. The molecule of water, OH_2, consists of two primaries associated in rotation. The first primary is an atom of oxygen, namely, two linked vortex rings. The second is a molecule of hydrogen, that is, two single vortex rings associated in rotation.

"The sulphur compounds afford good examples of molecules containing various numbers of primaries, that is we

have H_2S with two primaries H_2-S; SO_2 with three primaries S-O-O; and SO_3 with four primaries S-O-O-O" (J. J. Thomson, p. 122).

For an interesting discussion of the valency of nitrogen and of carbon, reference must be made to the fourth section of Thomson's book.

Dissociation. — It has already been stated that two rings will remain associated together in rotation only when their diameters are very nearly equal and their strengths also nearly the same. The degree of stability therefore of different molecules of the same substance will depend upon the nearness to equality of diameters of the component atoms in the given molecule, and the nearness to equality in the strength of these atoms. The degree of stability of different substances will depend upon similar conditions respecting their primaries, and also upon further conditions. Of these, the complexity of the molecule will be one; and others would arise, it would seem, from difference in beknottedness in the primaries or their components, and their differences in kinergety, and possibility of angular velocity of the primaries.

In a quantity of any given gas, the violence of the collision of molecules at any instant would be very diverse. It is not unreasonable to suppose that under any stated condition of temperature and pressure, some of these collisions would be sufficiently violent to split up the molecule. This would require merely that the impact upon one or more atoms of the molecule should be so severe as to change either its diameter or its velocity to a sufficient extent to cause it to move away from the other atoms with which it had been associated in the molecule. The component atoms thus separated from each other would remain separated until they came again into collision under the conditions necessary for reunion, or until they met with others with which they could combine. There would thus be throughout the

gas, under a stated condition, a certain definite proportion of atoms of disrupted molecules. This mechanical breaking up of molecules is called dissociation, and a gas in the condition just described is said to be partially dissociated.

The proportion of disrupted atoms would depend upon the average stability of the molecules of the gas, and upon the frequency of the collisions of great violence. Thus for different gases, the degree of dissociation at a stated temperature would depend upon the nature of the gas. For the same gas the degree of dissociation would increase with the temperature, and might also be to some extent a function of pressure. At a sufficiently high temperature, all or a sufficient number of the collisions might be so violent as to maintain the whole or nearly all of the gas in a state of dissociation. This condition would however be somewhat gradually rather than abruptly reached, as the temperature increased. These deductions from the theory tally very closely with the fundamental facts of dissociation. It would be interesting to see the theory further applied in connection with the modern extension of the dissociation idea of electrolysis and other portions of physical chemistry.

It is easily seen that a consideration of the conditions for the formation of molecules from atoms, and for the stability of the molecule when formed, must afford explanations of other observed chemical phenomena. Thomson briefly discusses the application of the theory to the time-rates of formation of chemical compounds, and to the fact that a given compound, once formed, is stable over a greater range of temperature than that over which its components can be made to unite directly to form the compound.

Gravitation. — The vortex-atom theory has been applied by Lord Kelvin to the explanation of gravitation, by pointing out its adaptability to the Le Sage theory. The possibilities of this theory when a perfectly elastic corpuscule is substituted for Le Sage's inelastic and perfectly hard one,

have been indicated at page 187. A system of vortex rings of the requisite fineness would fulfil all the apparent functions of the system of corpuscules, besides affording a fundamental explanation of their elasticity instead of leaving this in the position of an arbitrary postulate. Further, the vortex atom incidentally provides substances with precisely that openness of structure which Le Sage found essential to his theory but which seemed an otherwise uncalled-for and a rather fanciful suggestion.

The vortex ring then seems to lend itself to this theory of gravitation, which far exceeds in merit, any other yet advanced, and to which the most serious difficulty lies in the question of temperature equilibrium. The nature of this objection has been presented at page 188 for the case of elastic corpuscules having the character assumed for the molecule in the kinetic theory of gases. For the vortex corpuscule, somewhat the same objection awaits removal. Briefly stated, it is this: the corpuscular and atomic systems can, presumably, be in temperature equilibrium at only one temperature. The temperature of the corpuscular system must be constant. Hence there must be heating of some bodies and cooling of others by gravitation, which seems contrary to fact. But this objection, although presumptively good, has not been rigidly proved. As already remarked (cf. p. 189) the conditions of equilibrium under the peculiar circumstances of the case have not been duly investigated. Also there remain to be ascertained the consequences of the change of vortex diameter with temperature, and of the still unknown relation between vortex velocity and temperature. The theory is in a condition far from hopeless. Indeed, it appears to have arrived at that especially valuable stage, where further inquiry is definitely outlined, which if carried out is sure to be helpful whether it overturns or supports the whole fabric.

A consequence of the Le Sage theory, which appears to

have escaped notice, is that the weight of the atom cannot be proportional to its volume or mass; since the Le Sage corpuscules can act on the surface only of the atom. If then we apply this remark to the vortex atom, noting that the atoms are of the same average size, we see that the gravitational force between two atoms should be proportional to the product of their respective thicknesses. But, as their diameters are the same, their volumes are proportional to the squares of the thickness of the ring. Hence the gravitation force between two atoms must be proportional to the square root of the volumes of the atoms. If then we assume the fluid to be matter, then the weight, and therefore the kinergety of an atom is proportional to the square root of its mass. From this it would follow that mass is proportional to the square of kinergety, and not to kinergety directly as is ordinarily assumed (cf. p. 150).

Cohesion. — The force of cohesion in bodies is sensible only when the portions involved are apparently in exceedingly close proximity to one another. It is usually assumed to be a force which increases much more rapidly than the gravitation law of inverse squares would account for. This may however be only apparent, for if the atoms and molecules of bodies are composed of vortex rings, then as they approach one another, portions of the rings would be in contact or very near to one another, on the average, while the centers of the rings were still at relatively long distances. If, then, gravitation is due to a bombardment of the kind above supposed, the force holding two rings together when in contact at one point only, or nearly so, would be very much greater than the force of gravitation calculated from the ordinary law upon the basis of the distance between the centers of the rings. In other words, the distance law of gravitation upon this hypothesis may cease to hold at distances which are not many times the dimensions of atoms or molecules. It is quite possible, therefore, that the force

of cohesion may be due to the same action which produces gravitation; in other words, that the energy causing cohesion is identical with that causing gravitation.

Elasticity. — The elasticity of form of the vortex rings, arising solely from their energy of rotation, seems to afford a quite sufficient basis, in connection with the theory of cohesion, to account for the observed phenomena of elasticity in material bodies. Some possible applications of the theory to the explanation of these phenomena will be briefly suggested in a manner seemingly consistent with the usual presentation of the theory.

A single vortex ring when stretched along a diameter will have its transverse diameter shortened. It will oppose to the stretching force a resisting force proportionate to the amount of the distortion, if this be small. A similar statement holds for compression of a ring. A straight row of such rings, in contact with each other at the ends of diameters, and exposed to a compressive force acting along the row from each end, would exert an opposing elastic force proportional to the amount of compression. If such a row of rings were fastened together at their points of contact, and if each end ring were acted upon by a force tending to pull the rings apart in the direction of the row, the row would stretch by the distortion of the individual rings and would exert an elastic resisting force proportionate to the amount of stretch. The row would break apart when the stretching force exceeded the force holding together any two successive rings — the part played by the force of cohesion in material bodies. It is easy to see that a body made up of an aggregation of such rows of rings, and having an equal number of rows in every direction, would have some, at least, of the elastic properties of an isotropic solid. And similarly we may see that a body composed of vortex rings whose axes were scattered equally in all directions at random would likewise have some of the elastic properties of solids.

If the force holding the rings together, that is, the force of cohesion, were caused by a form of energy of the nature of that above ascribed to gravitation under the vortex theory, then the cohesive force would be insignificant except at the points of actual contact. It would diminish very rapidly in amount as the contact points were moved apart. Hence a force sufficient to overcome cohesion when the rings were actually in contact would be much more than sufficient to continually increase their distance, after separation. Whether a body composed of rings, not in actual contact with their neighbors, would exhibit the characteristics of a solid (*e.g.* rigidity) may be difficult to say. In such a body, the separation of the rings would presumably be maintained by their translatory motion and collisions. The effect of cohesive force in any such body would apparently be very small. The body would show little or no elastic resistance to extension, and its resistance to compression would be less than that of the same body with its rings in contact — very much less when the average distance between the rings became large in comparison with the diameter of the rings.

If, as it appears, the diameter of the vortex ring increases with the temperature, then the most compact aggregate of rings at a given temperature would still be capable of contraction with diminution of temperature, although capable of little or no reduction of volume by increase of pressure.

Remarks. — The very important chemical explanations arising as a direct result of this vortex theory, the remarkable extension of the applications of the theory already made, and their concordance with observed facts, indicate for this hypothesis far greater promise than attends the former, kinetic, theory or any of the other suggestions yet made. Its slow growth is clearly due to the rare skill required to deal competently with the mathematical problems

that it presents. If the theory has won less general acceptance than the careful study of it might seem to warrant, this is doubtless to be ascribed to the inevitable complexity of its mathematical treatment, and the want of an exposition which should clearly state the principles and results of the hypothesis in a manner intelligible to those not familiar with the higher branches of mathematics, or without the leisure to review the extensive literature of the subject. At a first glance, the fundamental conceptions involved in the theory may appear fanciful; but looking below the surface, we see, as has above been indicated, that this is distinctly contrary to the fact, since the properties assigned to the fluid imagined are of the simplest possible sort and with the exception of kinergety are negative in character. That no such fluid is known to us through any other means constitutes no objection to the hypothesis. There is no obvious reason why, if such a fluid does exist, we should be in any way aware of it except when it is in vortex motion. The fact that the motion of the rings cannot be philosophically accounted for is clearly not an objection, for it is no part of physics to account for the origin of matter or motion.

The following quotation gives the greater part of Maxwell's comment on the vortex-atom theory. It is taken from his article "Atom" in the Encyclopedia Britannica: —

"On the other hand, the vortex ring of Helmholtz, imagined as the true form of the atom by Thomson, satisfies more conditions than any atom hitherto imagined. In the first place, it is quantitatively permanent, as regards its volume and its strength, — two independent quantities. It is also quantitatively permanent as regards its degree of implication, whether 'knottedness' on itself or 'linkedness' with other vortex rings. At the same time, it is capable of infinite changes of form, and may execute vibrations

of different periods, as we know that molecules do. And the number of the essentially different implications of vortex rings may be very great without supposing the degree of implication of any of them very high.

"But the greatest claim to recognition of this theory, from a philosophical point of view, is that its success in explaining phenomena does not depend on the ingenuity with which its disciples 'save appearances' by introducing first one hypothetical force and then another. When the vortex atom is once set in motion, all its properties are absolutely fixed and determined by the laws of motion of the primitive fluid, which are fully expressed in the fundamental equations. . . .

[This] "primitive fluid has no other property than inertia, invariable density, and perfect mobility, and the method by which the motion of this fluid is to be traced is pure mathematical analysis. The difficulties of this method are enormous, but the glory of surmounting them would be unique.

"There seems to be little doubt that any encounter between two vortex atoms would be in its general character similar to those which we have already described. Indeed, the encounter between two smoke rings in air gives a very lively illustration of the elasticity of the vortex rings.

"But one of the first, if not the very first, desideratum in a complete theory of matter is to explain — first, mass, and second, gravitation. To explain mass may seem an absurd achievement. We generally suppose that it is of the essence of matter to be the receptacle of momentum and energy, and even Thomson, in his definition of his primitive fluid, attributes to it the possession of mass. But according to Thomson, though the primitive fluid is the only true matter, yet that which we call matter is not the primitive fluid itself, but a mode of motion of that primitive fluid. It is the mode of motion which constitutes the vortex ring,

and which furnishes us with examples of that permanence and continuity of existence which we are accustomed to attribute to matter itself. The primitive fluid, the only true matter, entirely eludes our perceptions when it is not endued with the mode of motion which converts certain portions of it into vortex rings, and thus renders it molecular.

"In Thomson's theory, therefore, the mass of bodies requires explanation. We have to explain the inertia of what is only a mode of motion, and inertia is a property of matter, not of modes of motion. It is true that the vortex ring at any given instant has a definite momentum and a definite energy, but to show that bodies built up of vortex rings would have such momentum and energy as we know them to have is, in the present state of the theory, a very difficult task."

In view of Professor J. J. Thomson's application of the hypothesis in the domain of chemical theory, one might be disposed to emphasize much more strongly the favorable part of this comment; for no other hypothesis has even attempted an explanation of chemical phenomena as a result of mere motion of matter, while Thomson's explanations are strikingly successful.

Maxwell's objection that the theory does not explain gravitation and mass may easily fail to command equally ready assent with his otherwise favorable view of the theory as a whole. The application of the theory to gravitation has been already briefly outlined; and whatever may be its shortcomings, it is clearly far in advance of all predecessors, and has not encountered disproof.

As to Maxwell's remark with regard to mass, it is not easy to entirely apprehend his position. He clearly interprets the term *inertia* as the equivalent of *mass*, and employs the latter in the sense in which *kinergety* has been

defined in these pages. Consistently with this he also specifically admits that the vortex atom in translatory motion possesses kinetic energy. He says: "It is true that the vortex ring at any given instant has a definite momentum and a definite energy, but to show that bodies built up of vortex rings would have such momentum and energy as we know them to have is, in the present state of the theory, a very difficult task." This comment can be understood only on the supposition that Maxwell regarded mass (kinergety) as an absolute quantity in the sense described in a later paragraph (cf. Part II. Chap. V.). If, however, as there pointed out, and as seems probable, energy is merely the power of one portion of matter to move another portion because in motion relatively to it, then kinergety is a relative quantity only. Remembering then that the "fluid" exerts no resistance to the motion of the rings, and that it is of uniform density in all rings, it is clear that the theory provides all the explanation of kinergety that any theory can be called upon to give. If the kinetic energy of a body is simply its power of imparting kinetic energy to a given body as compared with that of some other body, then we have only to account for the same relative power in the vortex atoms; and this is sufficiently done by assigning to them proper relative volumes and by assigning to the "fluid" either kinergety or a property from which this may arise. If the theory postulates only one property of matter it can hardly be called upon to explain that property.

But what is meant by the phrase "as we know them to have"? We form ideas of the mass or kinetic energy of a body in only three ways: by the acceleration of other bodies, by motion under or against forces, by the sensations accompanying acceleration or retardation of the motion of the object by our muscular action. The second of these gives us no information more than the first except in so far

as we associate with the idea of force the ideas derived from the sensations produced by force. But these are identical in kind with the sensations experienced in accelerating or retarding the motion of freely moving objects. Thus the three methods of forming impressions of the mass or kinetic energy of a body reduce to two. But the sensations in accelerating or retarding the free motion of objects (as also of exerting or resisting force) are merely our cognizance of the attendant displacement of portions of our muscular or nervous systems. And so far as the material action involved is concerned, this is merely the displacement of one aggregation of vortex atoms by another, which is all that we can say of the direct transfer of kinetic energy from one body to another. The sensations produced by light, heat, electrical currents, give no means of estimating mass or kinetic energy. In brief then we have no quantitative knowledge of "mass" (kinergety) or kinetic energy save through the power of displacing other bodies. We have therefore no knowledge of either as an absolute quantity. The theory has then to account only for the relative kinergeties of bodies; and therefore as the kinergety of the body is only the sum of the kinergeties of its atoms, to account for the relative kinergeties of the atoms. But the latter it does, as just stated. No absolute density need therefore be ascribed to the fluid. If there be such an intrinsic property as absolute density, it is no part of any physical theory to explain it. We here reach the confines of the domain of physics. It was not in this, however, that Maxwell criticised the theory, but only in its inability to account for the kinergety of the molecular aggregates called bodies, — a criticism which is untenable if the foregoing remarks are correct.

In this connection, a possible modification in the fundamental hypothesis of the vortex-atom theory may be noted. If as suggested at a later point (cf. Part II. Chap. V.),

kinergety can be adequately accounted for by the permanent and independent occupancy of space by each minutest portion of matter, then this simpler concept may be substituted for kinergety (or inertia or mass) as the postulated property of the perfect fluid. The theory then undertakes the grand task of developing analogues of all the properties of substances, and all forces and forms of energy from modes of motion of vortex rings in a hypothetical matter whose one assumed property is the permanent, continuous, and uniform occupancy of space, — a task which would appear hopeless and visionary had it not already been well begun.

Finally, as to its present status we may say that the theory has not yet been successfully assailed. This could be effected only by showing either that its wonderfully simple basal hypothesis is false, or that the mathematical deductions therefrom are defective or contradictory to fact. Neither has been done, nor has anything been proffered as a substitute for it. The theory has not yet, it is true, been found capable of satisfactorily accounting for several important classes of phenomena, for instance, the electro-magnetic, crystallic, and others; but this constitutes no valid disproof. One might thereby be led to despair of its ultimate success, especially in face of the serious mathematical difficulties in the way of its development; or one might abandon hope of a further outcome from it. But the value of the theory must be judged by what it has accomplished, not by what we have not yet succeeded in doing with it. And when thus tested, the theory still remains preëminent.

Of much interest are the following statements of the present attitude taken towards this theory by its two great exponents. The quotations are by permission from personal letters, in response to notes of inquiry which contained statements of views substantially as expressed in the preceding paragraphs.

"LONDON, 18th May, 1898.

"DEAR SIR, I am afraid it is not possible to explain all the properties of matter by the Vortex-atom Theory alone, that is to say, merely by motion of an incompressible fluid; and I have not found it helpful in respect to crystalline configurations, or electrical, chemical, or gravitational forces.

"I am afraid this is the only answer I can give to your letter of the 6th, which I have received to-day. I wish I could say a great deal more on a subject which has never ceased to interest me. We may expect that the time will come when we shall understand the nature of an atom. With great regret I abandon the idea that a mere configuration of motion suffices.

"Yours faithfully,

(Signed) "KELVIN.

"PROFESSOR S. W. HOLMAN,
"Brookline, Mass., U. S. A."

"6 SCROPE TERRACE, CAMBRIDGE,
"June 20, 1898.

"DEAR SIR, . . . With reference to the Vortex-atom Theory, I do not know of any phenomenon which is manifestly incapable of being explained by it; and personally I generally endeavor (often without success) to picture to myself some kind of vortex-ring mechanism to account for the phenomenon with which I am dealing. In lectures and papers, however, I generally content myself with an illustration which, though it has no claim to the fundamental character of one based on vortex motion, is easily conceived and handled by the mind, and so is more adapted as a guide to research.

"I regard, however, the vortex-atom explanation as the goal at which to aim, though I am afraid we know enough about the properties of molecules to feel sure that the distribution of vortex motion concerned is very complex.

"Believe me

"Yours very truly,

(Signed) "J. J. THOMSON.

"PROFESSOR S. W. HOLMAN."

CHAPTER V.

NATURE OF ENERGY AND MATTER.

Is all Energy Kinetic? — In kinetic energy we recognize two and only two factors or elements; namely, kinergety and velocity. The kinergety seems to be, or to arise from, an intrinsic property of the body or substance.

The only respect in which a body undergoes change as it gives up or receives kinetic energy, is in its velocity. The kinergety is unchanged.

If then the kinetic energy of a body A be expended without loss, in imparting kinetic energy to a body B, the only discoverable physical change in A or B or surroundings is the change in relative velocity of the two bodies. Moreover the only necessary antecedent condition is relative motion of the bodies.

But if a body A expends its kinetic energy in the production of heat in the body B, the change in A is absolutely in no wise different from that undergone when the result was kinetic energy in B; namely, a change in velocity. Or again, if the motion of A be directed upward, so that the kinetic energy is now transformed into gravitation energy, the change in A is still the same as before. The same is true for all other possible transformations of kinetic energy. Likewise in the transformation of any other form of energy into kinetic, the only change in the body in which the kinetic energy is produced is the change in velocity.

Now there seem to be three possible views of the character of the process called transformation. First, as in the transference of kinetic energy from one body to another

the only change in the receiving body is change of translatory velocity, so in the case of transformation the resulting change may likewise be only change in velocity,— although we may be unable to locate the portions of substance or matter undergoing this change of motion. Thus in the transformation of kinetic energy into heat, the heat energy produced may be due merely to increased velocity of random vibration of particles (molecules) of the heated body,— thus remaining kinetic in form, but molecular instead of molar. On account of the exceeding minuteness of the molecules, if they exist, we have been unable to obtain ocular or other direct demonstration of this hypothetical heat vibration; so that the kinetic or mechanical theory of heat is based entirely on speculation. Similarly, in the case above cited of transformation into gravitation energy, it is supposable, but not yet demonstrable, that the gravitation energy may consist in kinetic energy of a system of impinging particles which form no part of the substance of the body acted upon. Kinetic theories have been framed with more or less completeness for all forms of energy. The allusion to them here is not to raise the question of their acceptability, but to illustrate the degree of attention accorded to the view that all energy *may* be kinetic.

Second, we may suppose that some forms of energy are not kinetic in their nature, but are due to some condition other than motion, imposed upon the substance or matter of the body or its surroundings. The energy would then be due jointly to this condition and to kinergety, as the kinetic form is due to motion and kinergety. There appears to have been no suggestion of any such condition to account for any particular form; and the whole suggestion seems gratuitous and uncalled for, although it can hardly be declared impossible.

Third, we may suppose that energy is not kinetic at all; that is, that it is not due to motion, but is something which

is transferred from one body or place to another, and manifests itself by diverse phenomena, or in other words, assumes different forms, according to certain (unknown) conditions. In this view, transformation must be held to consist merely in so changing the conditions attending the bodies or the energy concerned that the energy shall manifest itself in another manner. The necessary procedure is known for a great variety of cases. But why it is that differences in manifestations should occur is wholly unknown and even unsurmised.

A conclusive selection between these three views unfortunately demands further knowledge or insight than has yet been brought to bear. The second view may be rejected, for reasons stated. Between the first and third, or as we may call them the kinetic and entity views, we may find ground for preference, but hardly for final decision. The weight of argument and the preponderance of sentiment appear to favor the kinetic view. From the foregoing presentation of the kinetic and entity views, a crucial test between them may be seen to lie in ascertaining whether motion alone is transferred when kinetic energy is changed into other forms. If so, then the kinetic view is established, and the principle of conservation of energy becomes merely a numerical relation. If something more than motion is transferred, then the kinetic view would fall, although the entity view would still require further demonstration. In kinetic energy itself, and in the processes of its transference and transformation, there seems to be absolutely nothing to suggest any change except in state or mode of motion, — nothing but changes of velocity. For evidence of anything more than this as the essential and complete fact in energy transformation, we must then turn to other forms of energy. But of the nature of these other forms we know absolutely nothing. They can therefore furnish no evidence either for or against; and sensation is equally at fault (p. 224). In

corroboration of these remarks it may be noted that all hypotheses of the ultimate nature of energy are kinetic. This, however, is not independent evidence, as it arises from the same causes as the other. There are two considerations which are perhaps contributive to the entity view. One of these is the entirely justifiable habit of regarding energy as a power transferable from one body to another. The other is the principle of conservation. These two ever-conspicuous ideas conjoined, easily produce a more or less vivid impression of energy as an entity, that is, as being an irresolvable thing transferable bodily, so to speak, from one object to another. This idea may be supported by a conservative view of radiation. "Radiant energy" is transmitted through portions of space apparently devoid of any of the elements, that is, of the elementary substances which we recognize through the possession of many different properties by each. Here therefore, it might be asserted, we are free to assume that energy exists apart from substance, or at least that such a position is more logical and conservative than to assume an otherwise unrecognizable substance, the ether, as a medium for transmission. We can but admit, however, that neither of the three considerations advanced affords any real support to the entity view. It is at least as natural to ascribe the "powers" to motion imposed on matter or substance as to regard it as an irresolvable entity; and as rational to regard the principle of conservation as a mere numerical law, as to assume that it implies an underlying entity. And as to the assumption of the ether as a transmission medium for radiant energy, this assumption is no whit more gratuitous than our unquestioned assumption of the existence of any one of our seventy elements. The only essential difference in the case lies in the fact that we find it necessary to ascribe to the ether fewer properties and these in a different degree than to the elements. Matter, as we have repeatedly insisted, is not

a thing recognized by the senses but only inferred or rather assumed as the vehicle of energy (cf. pp. 135, 159), and there is just the same *kind* of reason, for inventing the ether as for inventing any one of the elements. Deferring further detail as to the nature of the ether, enough has been given to show that there is here no definite support for the entity view of energy.

In more technical terms the argument is briefly this. In the change of a quantity of kinetic energy into some other form we have a quantity whose "dimensions" are ML^2T^{-2} equated to another quantity (say of gravitation energy), which must therefore have the dimensions also ML^2T^{-2}, as we can equate only quantities of the same dimensions. But in the transformation, the mass of the object giving up its kinetic energy is unchanged, and is in general unequal to the mass of the receiving body. The body which gives up its kinetic energy retains its mass or kinergety unchanged. Its change in condition is then only with respect to that which corresponds to the dimension L^2T^{-2}, — in this case, velocity squared. But if the change in the acting body is only in respect to L^2T^{-2} whereas the quantity conserved, that is, the energy, is of the dimensions ML^2T^{-2}; then, first, the energy must have been transferred to something which has the dimension M, that is, has kinergety and is therefore substance, or at least matter; second, the receiving substance will undergo a change in only such a condition or conditions as correspond to the dimensions L^2T^{-2}. If these inferences are correct, then we may conclude that energy cannot exist apart from that which has kinergety, and therefore apart from substance or at least matter, as we have defined these concepts. Further, if the dimension L^2T^{-2} can correspond to nothing but velocity squared, then all energy is kinetic. There is no apparent escape from the first of these conclusions; is there from the second?

There is then little to oppose to the view that all energy may be kinetic, but much to uphold it. At best, however, it can be regarded only as a working hypothesis. As such, it receives most effective support from the vortex-atom theory. The extreme simplicity of the kinetic view renders it of great service to certain minds by supplying concrete analogies to phenomena not otherwise clearly grasped. But it is ever needful to guard against the tendency to gradually install a pleasing analogy as a fixed fact. In the case of the hypothesis in hand, it must be remembered that we have no proof that any one form of energy other than that of molar motion is kinetic. Even for heat, so commonly classed unconditionally as molecular kinetic energy, no specific proof exists other than the general argument just given for all forms of energy. The experiments of Rumford and of Davy, for instance, however confirmed and generalized, while disproving the material nature of heat, do not prove that heat is due to a mode of motion. Irresistibly as they may be thought to indicate this view, the indication is no stronger or more direct than in *any* case of transformation of kinetic into any other form of energy. We may be permitted the reiteration that the sole, fundamental consideration of much weight is, that in the transformation of kinetic energy, the only discoverable change in the body expending or receiving that kinetic energy is change in velocity. Therefore, to assume that anything other than motion is transferred is gratuitous. But a quantity of molar kinetic energy is convertible directly or indirectly into an equal quantity of any other form, and *vice versa*. Hence the other forms must also be kinetic, or we must accept as the other horn of the dilemma the unsatisfactory if not untenable supposition that mere motion is convertible into something else or perhaps into several other things of whose nature there is not the slightest trace.

Still more concisely, the argument may be stated thus: When the kinetic energy of a body A is transferred, with change of form, to some other object or portion of space, the mass of A remains unchanged. The factor L^2/T^2 is the only one, therefore, which undergoes change in either A or B. But the quantity between which equality exists during the operation is of the dimension ML^2/T^2. Hence, first, the object or portion of space B must possess the dimension M, and therefore must be a substance, not a mere portion of space; in other words, energy cannot exist apart from matter. Second, unless a quantity which has the dimension L^2/T^2, and is the square of a velocity, can be changed into one which is not a velocity squared but has still the same dimensions, then all energy is kinetic.

The kinetic view of energy is not to be regarded as antagonistic to the practice now current, and apparently with good results, among certain German writers, of separating energy into factors (called capacity and intensity factors) which differ for different forms of energy. If energy is kinetic, the mode of motion from which each form arises is characteristic of that form, and the motion, in some forms, is probably not of the substance on which the manifestation of energy appears, but on some external substance. It is therefore to be anticipated that the intensity factors, even if they should prove ultimately to be all of the dimension L^2/T^2, would correspond to very different physical manifestations. For instance, suppose that heat energy were kinetic, then its intensity factor, temperature, should have the dimension L^2/T^2. Still, as we are able to perceive, not the individual molecular motions or their individual effects, but only the resultants of an immense number of these, it is clear that we should not expect our sense impressions or the physical phenomena of heat to be identical with those of the molar kinetic energy of isolated masses. The case of heat can differ only in degree from that of any other form

of energy, but in this one, our imaginations have been supplied with almost the sharp-sightedness of Maxwell's "demons," if not with their delicate alertness, by the kinetic theory of gases. It is to be remembered that the dimensions ML^2/T^2 may be separated into any number of different pairs of factors. Also it is not impossible that some of these pairs may represent actual physical quantities. Hence we may suppose, as the facts prove, that different forms of energy will be recognized through different factors, and that the same form may be recognized and measured also through different pairs of factors in some cases. The classification of the forms of energy in the three groups, space, surface, and volume energies, however useful, seems to the writer to be an outcome of this possible factoring rather than of intrinsic differences in the nature of the energy forms. And it is not unimportant to remark in this connection that the postulation of discontinuity, at the point of transformation of energy of one of these classes into that of another, involves the overlooking of an intermediate double transformation; namely, into and out of elastic energy. The further discussion of these points is, however, beyond the scope assigned to this book.

But whatever strength we may assign to the kinetic view of energy, we can hardly deny that the consensus of opinion of physicists is opposed to its introduction in the presentation of elementary physics beyond the position of an adjunct hypothesis. Confined to this function, it is certainly most useful, but as the substructure of physical teaching its employment is not only unwarrantable but would instil a most flagrant disregard of logic and of the scientific method.

Objection may be raised that this view of energy is inconsistent with the concept of matter as purely a carrier of energy, and of substance as endued with energy. There is, however, nothing in the definition to justify any such criticism. Matter or substance possessing energy in virtue of

an impressed mode of motion is as literally endued with, or a carrier of, energy as if the energy were a pack of iron.

On Chemical Energy. — On the nature of this form of energy, speculation has made less progress than for other forms, — possibly because the phenomena are complicated by the selective property of atomicity or quantivalence. But aided in our outlook by the vortex theory, we may find it easy and rational to dissociate, or at least to differentiate, atomicity from chemical energy. Atomicity may be wholly independent of chemical energy. It may be due to a purely structural characteristic of the atom in virtue of which two or more atoms can or cannot remain in stable association under the conditions which obtain in substances. Among these conditions we recognize but two as being essential factors in chemical action; namely, molecular impacts, and some force under which the atoms are held in combination, and under or against which chemical work is done. This we will call *chemical force* and the energy producing it *chemical energy;* what is their nature?

The function of chemical energy as thus differentiated is to maintain chemical force; to do the work attending chemical combination, thus providing the energy which appears as heat or in other ways in chemical processes; to account for the energy expended in chemical decompositions, etc. With atomicity referred to another cause than chemical energy, the latter manifests a selective power only through the differences in heat of combination of the elements. But this, as will be shown, might arise under a form of energy so universal and simple as gravitation in its law of action, through imposing the condition that the atoms should always approach each other through a fixed distance. Let us then proceed to develop the clue thus obtained.

Is chemical energy merely the manifestation of gravitation energy when the atomic distances are less than about the atomic diameters? The fundamental fact is this. At

distances large relatively to the atomic diameters, the atomic force, that is, the only perceptible force between any two atoms, is that known as gravitation, which follows Newton's law. At distances lying presumably between a very few diameters and a single diameter, the atomic or molecular force is called cohesion, using this term to include also adhesion. Of the relation of this force to mass and distance, almost nothing is known, owing to the exceeding minuteness of the portions of substance and of the distances with which the law deals. There appears to be nothing in the phenomena of cohesion, however, inconsistent with ascribing the force to the same energy which produces the force of gravitation. The distance law of cohesion is perhaps different from that of gravitation, but there is not the slightest evidence that the distance law of gravitation holds for the small distances at which alone cohesion is manifest. A concrete illustration of the progressive merging of one distance law into another is derivable, obviously enough, from the vortex-atom theory. Lastly, the distances at which the chemical force is manifested are almost certainly smaller than those at which the force is recognized as cohesion. There is no evidence that at these smallest distances, say less than a separation of a single diameter between centers, there are two distinct forces, chemical and cohesive, still less that there are three; namely, these two with gravitation. There seems to be no sufficient reason for not regarding the chemical force as merely the cohesion force at shorter atomic distances, with probably a still progressively changing distance law. In face of the radical divergence between chemical and cohesion phenomena this view would wear an air almost of absurdity were it not possible to point to some rational explanation or analogy for atomicity; but this has been done by Professor Thompson.

There remains then to be considered the heat of combination. Take first the specific case of vapor of sulphur

uniting with gaseous oxygen, under constant volume. Suppose the sulphur molecules to be monatomic, and uniformly diffused through the combining mass of oxygen. Then the combination would be that of S with O_2 to form SO_2 in the case of each molecule. Under energy acting as we have assumed, every molecule will experience a force towards every other molecule, but owing to the assumed uniformity in distribution, the resultant force on any molecule will be zero. As any molecule of sulphur approaches one of oxygen, their mutual force increases at first, possibly passing through a maximum, but being certainly large relatively to their force of gravitation at greater distances or to the kinergety of the molecules. A relatively large amount of positive work is thus done. A small negative quantity of work is also performed, since the average distance of the molecules from their neighbors is increased. The difference between these two quantities is the resultant work done by the chemical energy; and with the exception of a fraction perhaps left as vibration energy in the atoms, this work must result in an equivalent amount of heat. From the known magnitude of the cohesion forces and the closer juxtaposition of atoms in chemical combination than when we have to do with cohesion, it is clear that the work done in passing from the cohesion distance to the combination distance must be large. Whether it would be of an order of magnitude to equal the heat of combination can be ascertained only by numerical computation, for which unfortunately we have but crude data. This work must be shown to be not only sufficiently large but also sufficiently constant to correspond to the heat of combustion. As to its constancy, starting with the same initial condition of the combining substances, the average resultant force between a pair of approaching molecules (as S and O_2) will always be the same for the same distance between centers; also the average displacement under the force will be the same. Hence the work

done will be constant, as it should be. Moreover, a difference in initial condition of the substances will make relatively small change in the heat of combination, because the resultant force between the combining particles is small throughout the early part of their approach.

The above-cited simple instance of a gas uniting with another gas to form a gas, all at constant volume, can be easily extended to any combination by taking into account, in the well-known manner, the negative work attending any molecular separations occurring, and the external and other work incident to the operation.

For a more definite if less general presentation of this hypothesis as to chemical energy, let us use the terms of the vortex-atom theory, adopting also the modified Le Sage bombardment theory of gravitation. Then the one form of energy continuously in action causing approach or tendency to approach of all atoms, molecules, and bodies, is the energy of this system of bombarding vortex-corpuscules. This it is which is now suggested as constituting gravitation energy, cohesion energy, and chemical energy, and as exerting what is called chemical force, cohesive force, or gravitation force respectively, according to the degree of proximity of the atoms, molecules, or bodies involved. When in chemical combination, the vortex rings are supposed to be in the closest juxtaposition possible. Thus a molecule of HCl is supposed to consist of a single ring or atom of hydrogen and another of chlorine revolving about one another as described at page 206. The rings have their planes parallel, their diameters sensibly equal, and their distance apart very small relatively to their diameters. In other words, they are practically superposed, except for a separation minute as compared with the diameter of the ring. The Le Sage force here is the chemical force, and is large relatively to the kinergety of the atom; that is, under it the atom would acquire a high velocity in traversing a short distance. In all

molecules the atomic distances are thus small. In solids, even at their densest, the distances between centers of molecules presumably average more than the molecular diameter, and therefore more than the atomic diameter also. The Le Sage force between molecules at this distance, and up to somewhat greater ones, is, in general, the force of cohesion. Vortex molecules would be capable of somewhat closer packing than this, but the resulting solid would not be isotropic, and the molecules would have even less freedom than the small degree apparently pertaining to the molecules of a solid at ordinary temperatures. As the molecular distance increases slightly, the cohesion diminishes, and we have the analogy to the liquid state. And as the distance becomes several diameters, the Le Sage force passes gradually over from cohesion into gravitation, in which it presumably exists in perfect gases.

The distance law of the Le Sage force between a pair of simple vortex rings ought to be capable of mathematical expression. Starting with the two rings superposed, and drawing them apart perpendicularly to their planes, the force might perhaps lessen at first nearly in proportion to the distance between the planes. The rate of diminution would, however, increase with the distance, so that at a few diameters the law would approximate to that of gravitation to which it would conform with ever-increasing closeness as the distance grew larger. Suppose now that the rings be withdrawn from one another in a different manner. Let them be slid past one another in their own planes. At the start the Le Sage force will be very great, but it will diminish rapidly at first as the rings leave the position of immediate superposition, remaining rather large, however. It will diminish rather slowly as the rings slide over each other always in contact at two points, until the final point of outside contact is reached. The Le Sage force in this position should tally with or be somewhat greater than the force of

cohesion of solids. This appears probable from the closeness of the rings at the point of contact, and from the relatively small section of a vortex ring compared with its diameter.

The law of the Le Sage force as a function of the kinergeties of the rings, is presumably not entirely simple except, perhaps, for large distances; for the bombarding corpuscules impinge on the surface only of the rings, and the degree of implication of vortex atoms of different elements may be widely divergent. In the characteristic linkage or beknottedness of the atoms, the seat of the observed diversity of the cohesion of different substances is probably to be sought, as well as in the structure of the molecule.

Turning to a specific case, consider a mixture of hydrogen and oxygen gases. Any molecule H_2 and any molecule O_2 will find themselves under sensibly balanced forces during the greater portion of any free path; but as two such molecules approach within a few diameters, the resultant Le Sage force will become sensible and will rapidly increase. The molecules will therefore acquire increasing kinetic energy relatively to each other until, if moving in the right direction, they collide. From the impact they will rebound with altered velocities and vibrations. They cannot remain in association because the vortices H_2O_2 do not form a stable system, as shown at page 211; although it is conceivable that under very favorable and rare conditions they might for a brief time remain in combination as H_2-O-O (hydrogen dioxide), to be broken up at some shortly ensuing collision. The velocity and corresponding energy acquired by the molecules in approaching must be given up during recession, the energy being restored to the Le Sage system. The vibration energy will be subject to the Boltzmann restitution law (p. 177). Thus, on the average, no heat production or absorption will occur at an impact. This is the case at ordinary temperatures. At higher temperatures the vio-

lence of impact will be greater. Beyond a certain point dissociation will occur, as stated at page 214, some of the molecules O_2 or H_2 being separated into their constituent atoms O,O and H,H respectively. Some negative work will therein be done, heat being transformed into Le Sage energy. Suppose now an H_2 and a dissociated O to be approaching one another, as were the H_2 and the O_2 in the former case. If the direction of motion and the relative position of the vortex rings be unfavorable for combination, the H_2 and the O will rebound just as did the H_2 and the O_2. But if the conditions be favorable, the H_2 and O will remain in rotation about each other permanently; that is, the chemical combination will take place, and H_2O will be formed. During the approach, work will be done under the Le Sage force, here recognized as chemical force, and the translatory and vibratory energy of the system H_2O will be correspondingly increased. This translatory energy is heat, and the vibration energy will presently become heat. The Le Sage force between the H_2 and the O would presumably be less (probably one-half) at equal distances than between the H_2 and the O_2, as the O_2 has twice the mass and volume of the single atom O. But the closeness of ultimate approach in the combination would undoubtedly be greater in the case of the combination, and the whole work of approach results in heat. The observed heat of combination of a quantity of hydrogen and oxygen would then be the equivalent of the algebraic sum of the Le Sage work of dissociation and of combination, together with any other work incidentally performed. The elements of the action have been stated with perhaps unnecessary fulness. Of the application to more complex cases certainly nothing more need be said than that they involve only due consideration of all the work, both positive and negative, involved in the action.

By the introduction of the Le Sage force as a factor in chemical combination, a new element of stability is im-

parted to the molecule, but at the same time the violence of impact is also increased in a ratio nearly or quite corresponding; the necessity for the selective function of the vortex rings therefore still remains, as well to account for permanence as for valency. This extension of the Le Sage theory seems then to supplement the vortex-atom theory of chemical action in just the way needed to give it completion.

Without the aid of the vortex hypothesis, one could hardly arrive at the view of the nature of chemical action above presented, but this point once reached, we may easily see that the idea of the unity of chemical, cohesion, and gravitation energies does not depend on the tenability of that hypothesis. Almost any theory which would account for one of these three forms would presumably account also for the others, provided only that some consistent explanation for valency could be found. A view so comprehensive can but command interest even in so crude a form, and must doubtless have often suggested itself. Its development through the vortex hypothesis, if made, has escaped the author's attention.

The theory of chemical energy thus suggested can become entitled to weight only through mathematical and quantitative confirmation. This the author is unable to attempt beyond the following brief discussion. In view of this fact, the whole suggestion is given space only as a tentative speculation of small weight but of perhaps some interest.

Assuming the Newtonian law of gravitation to hold for the chemical force at all distances between atoms, we may readily deduce an expression for the work done by the chemical energy in causing the approach of atoms in chemical actions. Similar expressions may be derived for the action under other laws of force, but they need not be entered into here.

The work w, which will be done by the energy which

causes the approach of a pair of atoms, the distance law of the force being $F(s)$, will be,

$$w = \int F(s)\,ds.$$

If this energy be that of gravitation, and there be N pairs of molecules or radicals, one member of each pair consisting of n_1 atoms of atomic weightal a_1 and the other member having n_2 atoms of atomic weightal a_2, a_1 and a_2 being expressed in grammes, then the total work of approach W from a distance $s = d$ to a distance $s = x$ will be given by

$$W = n_1 a_1 \cdot n_2 a_2 \cdot CN^{-1} \int_x^d s^{-2} ds,$$

where C is the "gravitation constant," that is, the attraction between two "masses" of one gramme each at a distance of one centimeter between centers. The distances s, d, and x are measured between the centers of the atoms, and the vortex rings are assumed to have their planes parallel, and the line joining the centers is assumed to be perpendicular to these planes. The formula is not assumed to be universal, but is sufficiently general for the present purpose. In any chemical reaction, the heat evolved, expressed in ergs, will be the sum ΣW of all the separate quantities of combination work done, minus the work $\Sigma W'$ of separating the initial molecules into the radicals which unite to form the final molecules, and minus any external work W'' performed. That is, $HJ = \Sigma W - \Sigma W' - W''$. The quantity W' is of the same kind as W, and would be found from an expression similar to that for W.

As the distance x, the closeness to which the atoms must approach to permit of the performance of an amount of work consistent with the observed heats of combination in specific cases, is not known, we may apply the above expression to deducing its approximate value, from which we may then proceed to deduce an approximation to the order of

magnitude of the thickness of a vortex ring, as this cannot be greater than x. We shall make the assumption, justified by the sequel, that the chemical energy expended at distances greater than D is negligible.

The value of C is $6.5 \cdot 10^{-8}$ c.g.s. The value of N may be approximated as follows: The number of atoms of hydrogen in one gramme is 2 N, since the molecule is H_2. The gramme of liquid hydrogen at its boiling-point has a volume of about 14 cc. Assuming then that in liquids the molecules are arranged in square order, with centers at a distance D, or in an equivalent manner, $ND^3/2 = 14$, and $N = 28/D^3$. Adopting $D = 10^{-8}$, this gives $N = 2.8 \cdot 10^{25}$. As all molecules and atoms have roughly the same diameter, we may check this value by other liquids, *e.g.* water. Here N molecules have the volume, in the liquid state, of 18 cc.; so that $N = 18/D^3 = 1.8 \cdot 10^{25}$. Both numbers are presumably too small. We will adopt as a value $N = 3 \cdot 10^{25}$.

We will take the case of H_2O, formed from the direct combination of gaseous hydrogen and oxygen. As the approximation can at best be but rough, we may neglect W''. In this case, W' is the work of separation of O_2 into O,O. This is unknown, but is unlikely to be of a greatly different order of magnitude from W, that is, from the work $H_2 + O$, since the molecular operation is apparently of about the same scale of magnitude. Other considerations, such as the relative magnitudes of the heats of formation of H_2O and H_2O_2, and of CO and CO_2, seem to indicate that the order of magnitude of HJ, in simple cases like these, does not differ much from that of $\Sigma W'$. Hence we may neglect the unknown quantity $\Sigma W'$ in the present provisional computation. Then for H_2O under the distance law s^{-2}, we have, $HJ = 6.8 \cdot 10^4;\ 4.2 \cdot 10^7 = 2.9 \cdot 10^{12}$; $n_1 = 2$; $a_1 = 1$; $n_2 = 1$; $a_2 = 16$. Substituting the foregoing values in the above expression for HJ, and solving, we have, $x^{-1} - D^{-1} = 4 \cdot 10^{43}$. As $D = 10^{-8}$, D^{-1} is clearly negligible

compared with x^{-1}. Hence, approximately, $x = 2.5 \cdot 10^{-44}$ cm. Computations from HCl, CO, CO_2, and other compounds give exponents differing by only one or two units from this. The neglect of W′ can hardly be expected to introduce a greater error than a few units in this exponent. Hence we may suppose 10^{-44} to be about the order of magnitude of x, so that the thickness of a vortex ring is presumably not greater than 10^{-44} cm. Or, as $D = 10^{-8}$, $x/D = 10^{-36}$. If the force increases with the distance at a less rapid rate than under the Newtonian law, then the exponent will be much greater than 44, and the ring correspondingly thinner. Interesting lines of inquiry into the relative closeness of approach of atoms in different compounds, as to the heats of energy of combination of H + H, etc., and as to the distance law itself, naturally here suggest themselves.

The foregoing result at first inclines one to utter incredulity. A spider-web ring with a diameter equal to that of the earth's orbit would be of more substantial proportions. We were prepared, it is true, by one of the early deductions of the vortex-atom theory of chemical action to find the ring-thickness small as compared with the diameter, but we had hardly stretched the imagination to this figure. Still, only a moment's consideration is needed to show that this is precisely the condition absolutely demanded by the Le Sage theory of gravitation. To possess the degree of permeability demanded by the Le Sage theory, so that the force of gravitation shall not be modified by intervening portions of substance, or, in other words, so that Newton's law of masses shall hold universally within observed limits, the rings must be enormously thin relatively to their diameter. Let us attempt a rough numerical estimate of the lower limit of this permeability. We are probably justified in assuming that the gravitation between bodies on opposite sides of the sun is interfered with by the screening

action of the sun by less than one part in 10^6. Then, supposing the average distance between the centers of atoms to be one diameter, the shadow of a row of particles as long as the sun's diameter must be less than 10^{-6} of the area enclosed by a single ring. Assuming the rings most favorably arranged, this shadow could not exceed the sum of the shadows of so many separate rings. The row would consist of $1.5 \cdot 10^{11} \div 10^{-8} = 1.5 \cdot 10^{19}$. The shadow of this row of atoms must not exceed 10^{-6} of the area enclosed by a single ring. The width of the ring must then be less than 10^{-25} of the diameter. This rough estimate is almost certainly very much too large, but so far as it has any weight it sustains the deduction above made.

It is quite supposable that the conditions of stability of chemical compounds demand the same degree of thinness, but there seem to be no available means of applying that or any other test, and the discussion must rest at this point.

On the Ether. — The theory of the ether has received such careful and repeated exposition in connection with the phenomena of light and of electromagnetic radiation in general that a restatement here would be superfluous. A word may be said, however, as to the classification of ether relatively to substance and matter.

Stress has been laid in the foregoing pages on the point that substance is that which we *assume*, or at best *infer*, as existing in space, because we find there such quantities and manifestations of energy as indicate definite specific capacities for energy. Such capacities, differing permanently in degree, we are not able to reconcile with our concept of pure space; hence we impose on space, substance, to serve as the seat of these capacities and as the carrier of energy. It must be remembered that the capacities themselves are not directly observed, but are only inferred from the energy present and from its effects on other portions of substance or on our own senses. With these points in

mind, does it appear that the concept of the ether is in any way different in kind from that of substance? The answer to this query appears to be in the negative, so that the ether must be classed as either substance or matter. The ether is primarily that which is inferred as existing in space to serve as the carrier of radiant energy. Radiant energy is transmitted through it at a rate determined by some property of the ether and not by the properties and conditions of the radiating body solely. This fact of a constant velocity of transmission, — the same for radiations of the most diverse character and intensity, — supplies the same warrant in kind for adopting the assumption of the existence of ether that we have for adopting that of any of the elements. The difference, fundamentally and apart from hypothesis as to mode of propagation, lies solely in the small number of energy forms which we are obliged to find capacities for in the ether. But the apparent absence of gravitation, of chemical action, and so on, is no more a reason for the exclusion of the ether from the list of substances than obtains between different recognized substances through lack of magnetic capacity, and other differences. And if we turn to the theories of energy propagation in the ether, we find that they impose upon the ether the fundamental properties of substance; namely, kinergety and elasticity. From the point of view adopted in this book, then, there is no sufficient reason for not assigning to the ether a place as substance. Its precise relation to the elements need give us no concern until it is found taking part in chemical action. Whether its elasticity arises after the manner of that of substances, or in some other way, is a question whose solution seems almost hopelessly remote. It seems however to be admitted that a grained structure of the ether might not be inconsistent with its observed function, provided that the granulation were of a sufficient order of fineness. Be these things as they may, however, there appears

to be no reason for the traditional isolation of the ether as a concept quite apart from that of substance, and utterly foreign to it, — as a something vague and mystical, eventually to step forth from fairyland and furnish the explanation of all the properties of "gross" substances.

On the Nature of Force. — The mode of action by which energy exerts force varies presumably with the form of energy exerting the force. As we know nothing of the nature of these sundry forms save by hypothesis, so our knowledge of their actions, called forces, can be only of a hypothetical character. The views which have been presented incidentally to the discussions of the forms of energy will be briefly summarized here.

The most elementary of the forces seems to be that of elasticity. We trace this back to the molecules or ultimate particles of substances. There, we have but one rational hypothesis to account for it in terms of something apparently more simple. This refers it to the tendency of vortex rings to resume their normal shape when distorted, as through collision. This tendency is due to the energy of the vortex motion; and the energy which the distorted ring can give out is the translatory energy unequally distributed through it as a result of the impact of other rings with which it is or has been in collision. Further, as the imparting of motion from one ring to another can take place only through simple displacement of portions of the inert inelastic "fluid," or matter, by another portion, elastic force reduces ultimately to mere displacement.

Under the hypothesis herein advanced, gravitation, cohesion, and even chemical force, are supposed to be due to a bombardment process analogous to that so generally accepted as accounting for the pressure of gases. In the process, however, an essential and fundamental element is seen to be the exertion of elastic force by the molecules or particles of the systems involved. It is indeed this which

constitutes the observed force, the bombardment being merely the process by which it is called into play.

All explanations of electrical, magnetic, and electromagnetic forces, and of all forces involved in radiation, likewise reduce in the end to elastic force; and the list of forces is thus exhausted.

Thus we easily arrive at the hypothesis that the force actually exerted by all forms of energy is really of one kind; namely, elastic force. The difference between the various forces, then, consists only in the different modes by which the several forms of energy bring into play this elastic force. A little reflection will show that this view may be arrived at from rather more general grounds than the above; so that it has really more weight than some of the hypotheses incidental to the foregoing deduction.

If now we revert to elastic force under the aspect above presented, we arrive at the *Ultima Thule* of speculation in the deduction that all force is in the end mere displacement of the continuous hypothetical matter. Such speculations afford a unity and breadth of view of physical phenomena not otherwise attainable, and in so far are good. But their usefulness depends largely on the rationality of the process by which they are deduced, and on the firmness with which their purely hypothetical character is held in mind.

On the Nature of Matter and Kinergety. — In continuation of the discussion of Part I., Chapter XI., it may be remarked that if the sole property of *substance* for which we are to find a counterpart in *matter*, or which we are to transfer to it is kinergety, then speculation on the nature of matter resolves itself into speculation as to the nature of kinergety.

When we push against an object such as a table resting firmly on the floor, and the object does not move, we feel a resistance to the hands. We know that this arises from a resistance to motion manifested by the table, due ultimately to a frictional resistance between the table

and the floor. Even if the table slides, we feel a like resistance, and know that it is due to the same friction, at least in part. But if the table were mounted on a frictionless level railway, we should experience in accelerating its motion a resistance entirely indistinguishable in kind from that encountered in the other cases. The motion would be different, but the sensation itself would be the same. In starting into motion a cannon ball suspended by a long cord we should experience a like resistance. As there is no external resistance in the last two, the assumption is not unnatural that the object itself exerts a resistance. And this statement is true in so far as it is interpreted to mean only that the elastic energy in the body exerts a resistance against the hands. It is not true, as has been shown (cf. p. 140), of the matter composing the table. Nevertheless it is undoubtedly by supposing matter even when free to be capable of resisting motion, that there is commonly ascribed to matter, quite unconsciously, an attribute which from its vagueness and unreality is difficult to put into words.

This vague attribute is a property in virtue of which matter is supposed to be able to resist motion or when in motion to resist retardation. By some persons it is associated with the idea of *inertia*, by others with that of *mass*. It properly belongs with neither of these. If it corresponds to any real physical fact at all, that fact must be related to the property of kinergety. As matter is non-resistant, the above process of demonstrating this property is incorrect, and therefore the attribute may be illusory. On the other hand, "kinergety," and "the capacity for kinetic energy," are but names for a property of substance for which we seek naturally some intelligible explanation in matter. This might lead us back to the same vague attribute as before, which as nearly as it can be expressed, perhaps, is implied by such terms as substantiality, denseness, solidness, and the

like. The idea may also possibly be suggested by the phrase absolute density, and if so, that phrase may at the same time serve the added purpose of pointing the inconsistency of the attribute with modern physical science where the relative so completely displaces the absolute.

Now our sensation of force exerted on us from without is our cognizance of the displacement of some portion of our muscular or nervous system and of the effort exerted to resist that displacement. Thus our sense recognition of kinergety depends on the power of one portion of substance to displace another portion relatively to which it is in motion; namely, some part of our body. We therefore learn nothing further of the nature of kinergety from sensation than we already know from the action of inert substance. Hence whether the supposed property of "absolute density" inferred in the preceding paragraph is or is not real, we have no ground for it in sensation.

It appears then that all that is to be accounted for in kinergety is the power of one portion of substance to displace another relatively to which it is in motion. A substance of which every minutest portion permanently occupies its own quota of space, would seem to possess this property. Unless however these minutest particles were in every way indistinguishable from one another they would be thereby shown to have some other remaining property. But we have supposed that matter has no other recognizable property than kinergety. Therefore these minutest particles or rather portions must be indistinguishable. Continuity is also demanded in that out of which substances are made up, as otherwise, being devoid of force, its aggregations into particles of substance could not be permanent. Continuous, uniform, and permanent occupancy of space appears then to be the ultimate and sole property demanded of matter in order that it should fulfil all that we know of it.

The very natural objection to such a view of matter as

that just presented is, of course, that we are unable to satisfactorily grasp such a concept. But the difficulty is hardly greater than for the abstract time and space concepts or for all fundamental abstract ideas. We may be unable to picture to ourselves mere permanent occupancy of space, but no more are we able to imagine what can constitute absolute density, "mass," "inertia," or whatever other substitute we may be accustomed to think of as the ultimate property of matter; moreover we have here to do with a psychological rather than a physical point. It is in no respect more difficult. We must not overlook the fact, however, that the view is merely the result of pushing speculation to the limit, so that it is at best only of tentative character; and also that its acceptance or rejection is not necessary to a competent knowledge of physics. Still, the unity of physical ideas of which this view forms the key-stone is not without its value. Briefly it is as follows:—

Matter: that which is assumed to uniformly and continuously fill infinite space. Atoms: permanent aggregations of matter, differentiated from matter by some mode of motion (*e.g.* vortex motion), and from each other by structural characteristics (*e.g.* volume, size, shape, etc.). Molecules: congeries of atoms. Forms of Energy: due to modes of motion of the atoms or molecules themselves, or of special atoms or corpuscules impinging on them. Force: the action of one or more forms of energy, producing tendency to motion in the molecules or atoms, or their aggregates acted upon. Work: the complete action of energy when this motion takes place. Substance: atoms, molecules, or molecular aggregates. Bodies and Objects: limited portions of substance. The Ether: presumably a substance of very special and few properties. Properties: characteristic of substances, derived from atomic structure or from the action of energy.

INDEX.

8

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